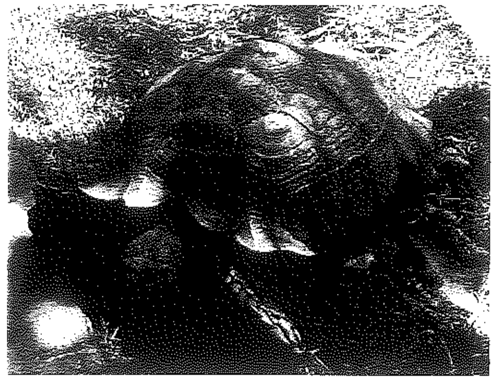
1.3 Tracking the Tortoise

A Solidify Understanding Task



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You may remember a task from last year about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race.

In the task, we modeled the distance from the starting line that both the tortoise and the hare travelled during the race. Today we will consider only the journey of the tortoise in the race.

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d(t) = 2^t$ (d in meters and t in seconds)

The tortoise family decides to watch the race from the sidelines so that they can see their darling tortoise sister, Shellie, prove the value of persistence.

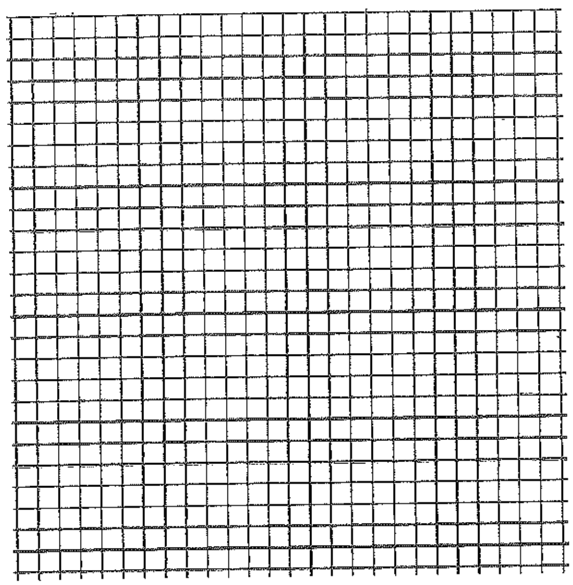
1. How far away from the starting line must the family be, to be located in the right place for Shellie to run by 5 seconds after the beginning of the race? After 10 seconds?

2. Describe the graph of *d(t)*, Shellie's distance at time *t*. What are the important features of *d(t)*?

3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past?

4. How long must they wait to see Shellie run by if they stand 1024 meters away from her starting point?

5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point.

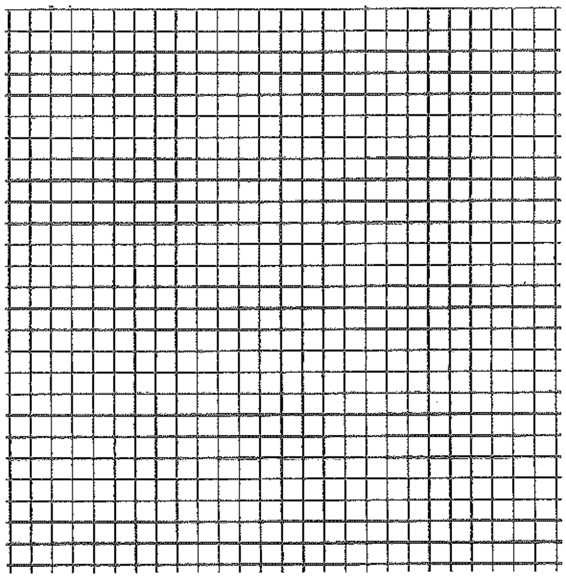


6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point?

7. What is the relationship between $d(t)$ and the graph that you have just drawn? How did you use $d(t)$ to draw the graph in #5?

8. Consider the function $f(x) = 2^x$.
   A) What are the domain and range of $f(x)$? Is $f(x)$ invertible?

   B) Graph $f(x)$ and $f^{-1}(x)$ on the grid below.



   C) What are the domain and range of $f^{-1}(x)$?

9. If $f(3) = 8$, what is $f^{-1}(8)$? How do you know?

10. If $f\left(\frac{1}{2}\right) = 1.414$, what is $f^{-1}(1.414)$? How do you know?

11. If $f(a) = b$ what is $f^{-1}(b)$? Will your answer change if $f(x)$ is a different function? Explain.

READY, SET, GO! Name Period Date

READY

Topic: Solving exponential equations.

Solve for the value of *x*.

1. $5^{x+1} = 5^{2x-3}$
2. $7^{3x-2} = 7^{-2x+8}$
3. $4^{3x} = 2^{2x-8}$
4. $3^{5x-4} = 9^{2x-3}$
5. $8^{x+1} = 2^{2x+3}$
6. $3^{x+1} = \frac{1}{81}$

SET

Topic: Exploring the inverse of an exponential function

In the fairy tale *Jack and the Beanstalk*, Jack plants a magic bean before he goes to bed. In the morning Jack discovers a giant beanstalk that has grown so large, it disappears into the clouds.

But here is the part of the story you never heard. Written on the bag containing the magic beans was this note.

Plant a magic bean in rich soil just as the sun is setting. Do not look at the plant site for 10 hours. (This is part of the magic.) After the bean has been in the ground for 1 hour, the growth of the sprout can be modeled by the function $b(t) = 3^t$. (b in feet and t in hours)

Jack was a good math student, so although he never looked at his beanstalk during the night, he used the function to calculate how tall it should be as it grew. The table on the right shows the calculations he made every half hour.

Hence, Jack was not surprised when, in the morning, he saw that the top of the beanstalk had disappeared into the clouds.

Time (hours)	Height (feet)
1	3
1.5	5.2
2	9
2.5	15.6
3	27
3.5	46.8
4	81
4.5	140.3
5	243
5.5	420.9
6	729
6.5	1,262.7
7	2,187
7.5	3,788
8	6,561
8.5	11,364
9	19,683
9.5	34,092
10	59,049

7. Demonstrate how Jack used the model $b(t) = 3^t$ to calculate how high the beanstalk would be after 6 hours had passed. (You may use the table but write down where you would put the numbers in the function if you didn't have the table.)

8. During that same night, a neighbor was playing with his drone. It was programmed to hover at 243 ft. How many hours had the beanstalk been growing when it was as high as the drone?

9. Did you use the table in the same way to answer #8 as you did to answer #7? Explain.

10. While Jack was making his table, he was wondering how tall the beanstalk would be after the magical 10 hours had passed. He quickly typed the function into his calculator to find out. Write the equation Jack would have typed into his calculator.

11. Commercial jets fly between 30,000 ft. and 36,000 ft. About how many hours of growing could pass before the beanstalk might interfere with commercial aircraft? Explain how you got your answer.

12. Use the table to find $f(7)$ and $f^{-1}(11,364)$.

13. Use the table to find $f(9)$ and $f^{-1}(9)$.

13. Explain why it's possible to answer some of the questions about the height of the beanstalk by just plugging the numbers into the function rule and why sometimes you can only use the table.

GO

Topic: Evaluating functions

The functions $f(x), g(x), and\ h(x)$ are defined below.

$f(x) = -2x$ $g(x) = 2x + 5$ $h(x) = x^2 + 3x - 10$

Calculate the indicated function values. Simplify your answers.

14. $f(a)$
15. $f(b^2)$
16. $f(a + b)$
17. $f(g(x))$
18. $g(a)$
19. $g(b^2)$
20. $g(a + b)$
21. $h(f(x))$
22. $h(a)$
23. $h(b^2)$
24. $h(a + b)$
25. $h(g(x))$