1.4 Pulling a Rabbit Out of the Hat

A Solidify Understanding Task

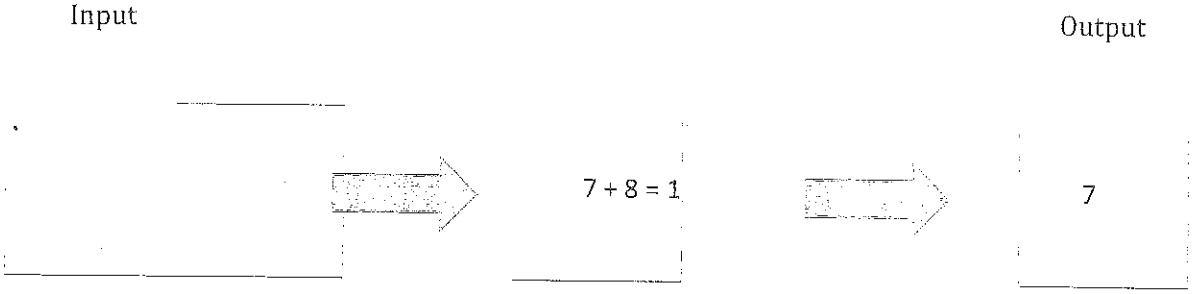
I have a magic trick for you:

- Pick a number, any number.
- Add 6
- Multiply the result by 2
- Subtract 12
- Divide by 2
- The answer is the number you started with!

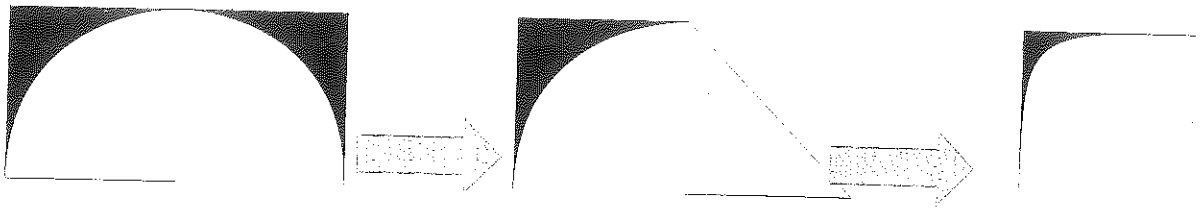
People are often mystified by such tricks but those of us who have studied inverse operations and inverse functions can easily figure out how they work and even create our own number tricks. Let's get started by figuring out how inverse functions work together.

For each of the following function machines, decide what function can be used to make the output the same as the input number. Describe the operation in words and then write it symbolically.

Here's an example:



1.



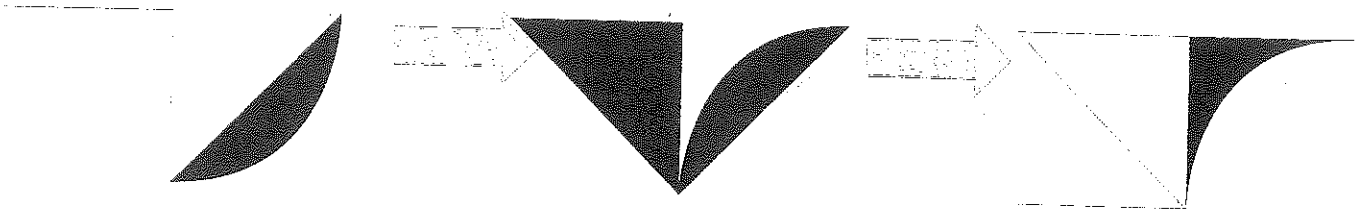
$f(x) = 3x$

$(x) =$

2.

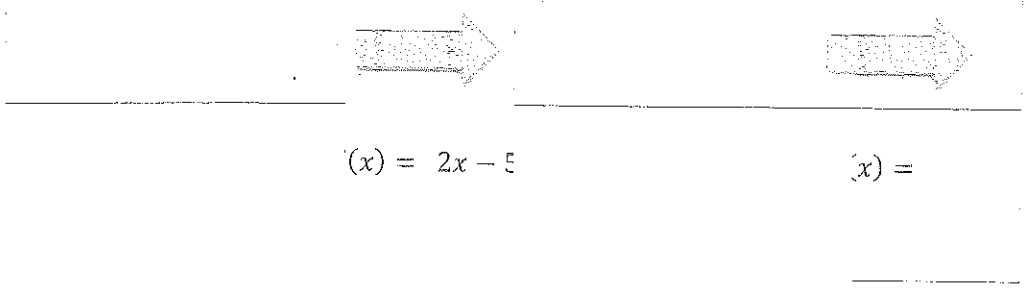
ords:

3.



words:

4.



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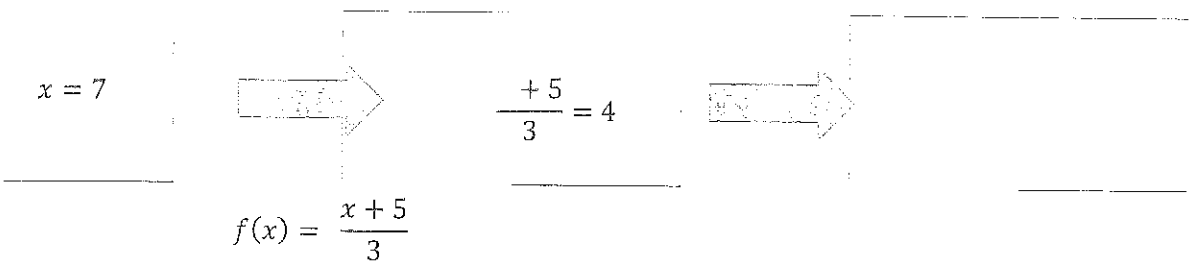

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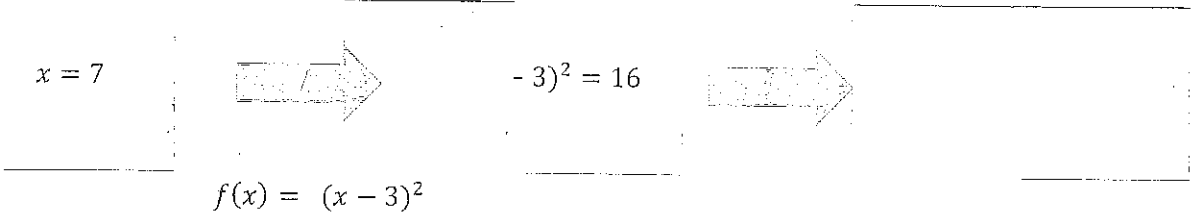

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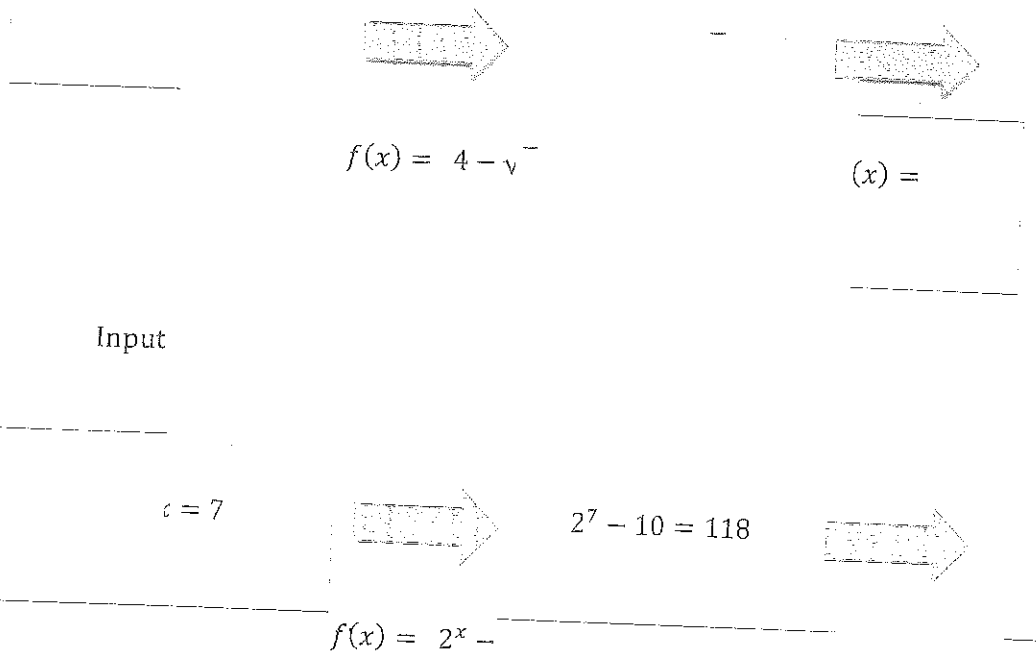


9. Each of these problems began with $x = 7$. What is the difference between the x used in $f(x)$ and the x used in $f^{-1}(x)$?

10. In #6, could any value of x be used in $f(x)$ and still give the same output from $f^{-1}(x)$? Explain. What about #7?

11. Based on your work in this task and the other tasks in this module what relationships do you see between functions and their inverses?

1.4

READY, SET, GO!

Name Period Date

READY

Topic: Properties of exponents

Use the product rule or the quotient rule to simplify. Leave all answers in exponential form with only positive exponents.

1. $3^6 \cdot 3^5$
2. $7^2 \cdot 7^6$
3. $10^{-4} \cdot 10^7$
4. $5^9 \cdot 5^{-6}$
5. $p^2 p^5$
6. $2^6 \cdot 2^{-3} \cdot 2$
7. $b^{11} b^{-5}$
8. $\frac{7^5}{7^2}$
9. $\frac{9^8}{9}$
10. $\frac{3^5}{3^8}$
11. $\frac{7^{-4}}{7^{-8}}$
12. $\frac{p^{-3}}{p^5}$

SET

Topic: Inverse function

13. Given the functions $f(x) = \sqrt{x} - 1$ *and* $g(x) = x^2 + 7$:

    a. Calculate $f(16)$ *and* $g(3)$.

    b. Write $f(16)$ as an ordered pair.

    c. Write $g(3)$ as an ordered pair.

    d. What do your ordered pairs for $f(16)$ and $g(3)$ imply?

    e. Find $f(25)$.

    f. Based on your answer for $f(25)$, predict $g(4)$.

    g. Find $g(4)$. Did your answer match your prediction?

    h. Are $f(x)$ *and* $g(x)$ inverse functions? Justify your answer.

Match the function in the first column with its inverse in the second column.

$f(x)$	$f^{-1}(x)$
16. $f(x) = 3x + 5$	a. $f^{-1}(x) = log_5 x$
17. $f(x) = x^5$	b. $f^{-1}(x) = \sqrt[3]{x}$
18. $f(x) = \sqrt[5]{x-3}$	c. $f^{-1}(x) = \frac{x-5}{3}$
19. $f(x) = x^3$	d. $f^{-1}(x) = \frac{x}{3} - 5$
20. $f(x) = 5^x$	e. $f^{-1}(x) = log_3 x$
21. $f(x) = 3(x + 5)$	f. $f^{-1}(x) = x^5 + 3$
22. $f(x) = 3^x$	g. $f^{-1}(x) = \sqrt[5]{x}$

GO

Topic: Composite functions and inverses

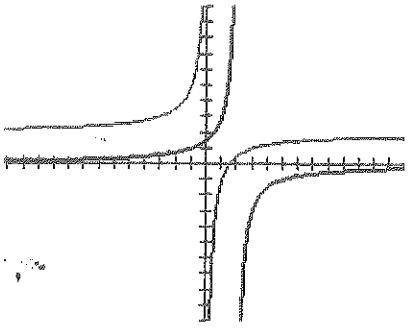
Calculate $f(g(x))$ and $g(f(x))$ for each pair of functions.

(Note: the notation $(f \circ g)(x)$ and $(g \circ f)(x)$ means the same thing as $f(g(x))$ and $g(f(x))$, respectively.)

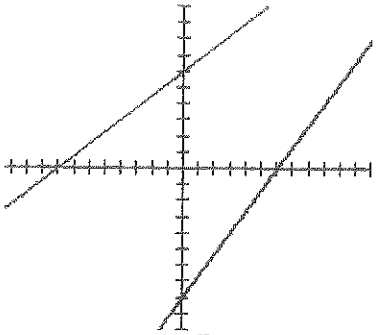
23. $f(x) = 2x + 5$ $\quad g(x) = \frac{x-5}{2}$	24. $f(x) = (x + 2)^3$ $\quad g(x) = \sqrt[3]{x} - 2$
25. $f(x) = \frac{3}{4}x + 6$ $\quad g(x) = \frac{4(x-6)}{3}$	26. $f(x) = \frac{-3}{x} + 2$ $\quad g(x) = \frac{-3}{x-2}$

Match the pairs of functions above (23-26) with their graphs. Label $f(x)$ and $g(x)$.

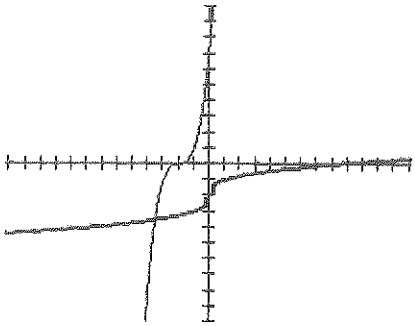
a.



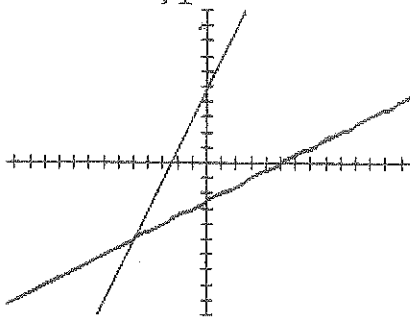
b.



c.



d.



27. Graph the line y = x on each of the graphs above. What do you notice?

28. Do you think your observations about the graphs in #27 has anything to do with the answers you got when you found $f(g(x))$ and $g(f(x))$? Explain.

29. Look at graph *b*. Shade the 2 triangles made by the y-axis, x-axis, and each line. What is interesting about these two triangles?

30. Shade the 2 triangles in graph *d*. Are they interesting in the same way? Explain.