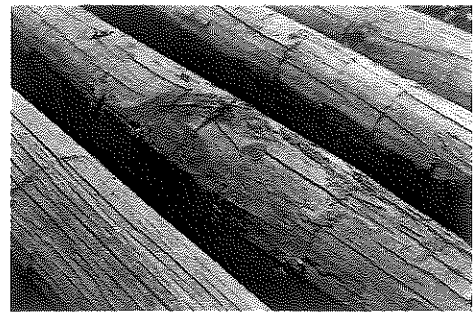
2.1 Log Logic

A Develop Understanding Task



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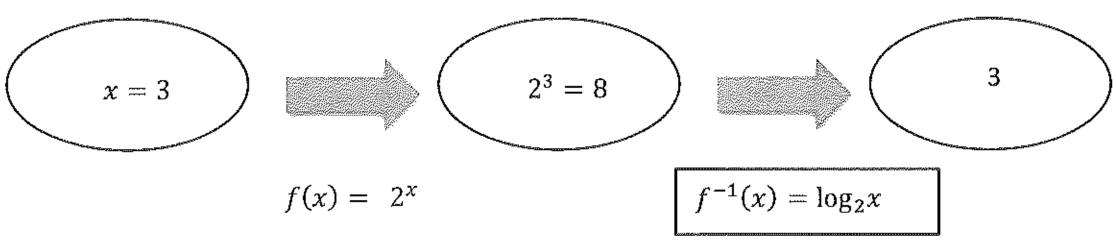
We began thinking about logarithms as inverse functions for exponentials in *Tracking the Tortoise.* Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:



We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\log_{10} 100 = 2$

$10^3 = 1000$ so, $\log_{10} 1000 = 3$

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The notation may see different, but you can see the inverse pattern where the inputs and outputs switch.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns related to logarithms.

Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

1. A. $\log_3 3$ B. $\log_3 9$ C. $\log_3 \frac{1}{3}$ D. $\log_3 1$ E. $\log_3 \frac{1}{9}$

Explain: ____

2. A. $\log_3 81$ B. $\log_{10} 100$ C. $\log_8 8$ D. $\log_5 25$ E. $\log_2 32$

Explain: ____

3. A. $\log_7 7$ B. $\log_9 9$ C. $\log_{11} 1$ D. $\log_{10} 1$

Explain: ____

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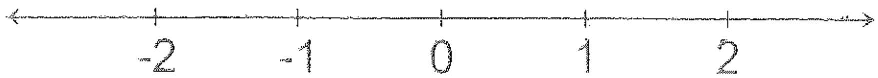
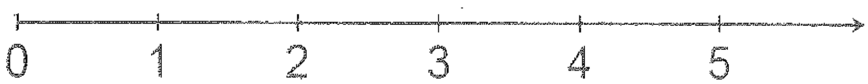
4. A. $\log_2\left(\frac{1}{4}\right)$ B. $\log_{10}\left(\frac{1}{1000}\right)$ C. $\log_5\left(\frac{1}{125}\right)$ D. $\log_6\left(\frac{1}{6}\right)$



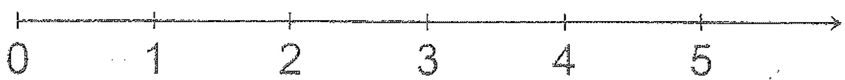
Explain: ____

5. A. $\log_4 16$ B. $\log_2 16$ C. $\log_8 16$ D. $\log_{16} 16$



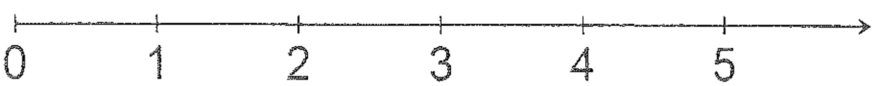
Explain: ____

6. A. $\log_2 5$ B. $\log_5 10$ C. $\log_6 1$ D. $\log_5 5$ E. $\log_{10} 5$



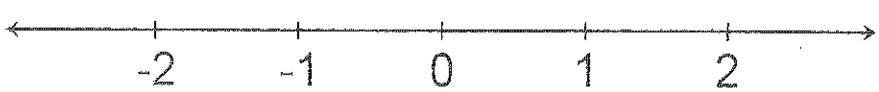
Explain: ____

7. A. $\log_{10} 50$ B. $\log_{10} 150$ C. $\log_{10} 1000$ D. $\log_{10} 500$



Explain: ____

8. A. $\log_3 3^2$ B. $\log_5 5^{-2}$ C. $\log_6 6^0$ D. $\log_4 4^{-1}$ E. $\log_2 2^3$



Explain: ____

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Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of $\log_b x$ is positive.

Explain: ____

10. $\log_b x$ is not a valid expression if x is a negative number.

Explain: ____

11. $\log_b 1 = 0$ for any base, b > 0.

Explain: ____

12. $\log_b b = 1$ for any b > 0.

Explain: ____

13. $\log_2 x < \log_3 x$ for any value of x.

Explain: ____

14. $\log_b b^n = n$ for any b > 0.

Explain: ____

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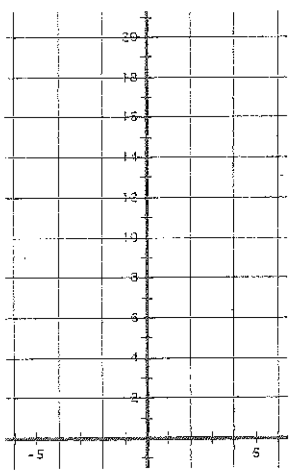
READY, SET, GO! Name Period Date

READY

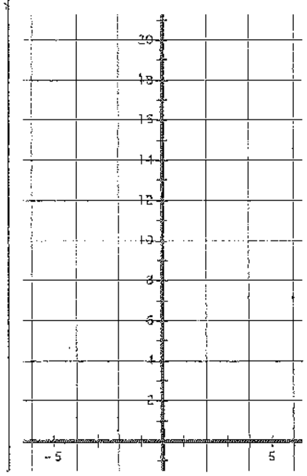
Topic: Graphing exponential equations

Graph each function over the domain $\{-4 \le x \le 4\}$.

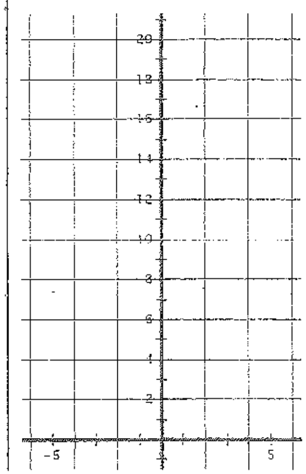
1. $y = 2^x$



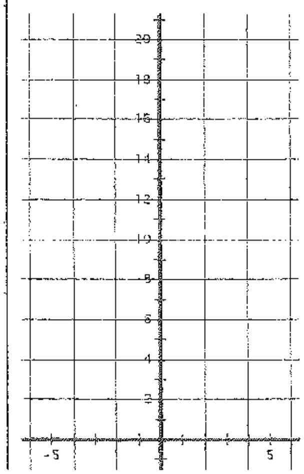
2. $y = 2 \cdot 2^x$



3. $y = \left(\frac{1}{2}\right)^x$



4. $y = 2\left(\frac{1}{2}\right)^x$



5. Compare graph #1 to graph #2. Multiplying by 2 should generate a dilation of the graph, but the graph looks like it has been translated vertically. How do you explain that?

6. Compare graph #3 to graph #4. Is your explanation in #5 still valid for these two graphs? Explain.

SET

Topic: Writing the logarithmic form of an exponential equation.

Definition of Logarithm: *For all positive numbers a, where a ≠ 1, and all positive numbers x,*

$$y = \log_a x \text{ means the same as } x = a^y.$$

*(Note the **base** of the exponent and the **base** of the logarithm are both **a**.)*

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7. Why is it important that the definition of logarithm states that the base of the logarithm does not equal 1?

8. Why is it important that the definition states that the base of the logarithm is positive?

9. Why is it necessary that the definition states that x in the expression $log_a x$ is positive?

Write the following exponential equations in logarithmic form.

Exponential form	Logarithmic form	Exponential form	Logarithmic form
10. $5^4 = 625$		11. $3^2 = 9$	
12. $\left(\frac{1}{2}\right)^{-3} = 8$		13. $4^{-2} = \frac{1}{16}$	
14. $10^4 = 10000$		15. $a^y = x$	

16. Compare the exponential form of an equation to the logarithmic form of an equation. What part of the exponential equation is the answer to the logarithmic equation?

Topic: Considering values of logarithmic functions

Answer the following questions. If yes, give an example or the answer. If no, explain why not.

17. Is it possible for a logarithm to equal a negative number?

18. Is it possible for a logarithm to equal zero?

19. Does $log_x 0$ have an answer?

20. Does $log_x 1$ have an answer?

21. Does $log_x x^5$ have an answer?

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GO

Topic: Reviewing properties of Exponents

Write each expression as an integer or a simple fraction.

22. 27^0

23. $11(-6)^0$

24. -3^{-2}

25. 4^{-3}

26. $\frac{9}{2^{-1}}$

27. $\frac{4^3}{8^0}$

28. $3\left(\frac{29^3}{11^5}\right)^0$

29. $\frac{3}{6^{-1}}$

30. $\frac{32^{-1}}{4^{-1}}$

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