2.4 Log-Arithm-etic

A Practice Understanding Task


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Abe and Mary are feeling good about their *log* rules and bragging about their mathematical prowess to all of their friends when this exchange occurs:

Stephen: I guess you think you're pretty smart because you figured out some *log* rules, but I want to know what they're good for.

Abe: Well, we've seen a lot of times when equivalent expressions are handy. Sometimes when you write an expression with a variable in it in a different way it means something different.

1. What are some examples from your previous experience where equivalent expressions were useful?

Mary: I was thinking about the *Log Logic* task where we were trying to estimate and order certain *log* values. I was wondering if we could use our log rules to take values we know and use them to find values that we don't know.

For instance: Let's say you want to calculate $\log_2 6$. If you know what $\log_2 2$ and $\log_2 3$ are then you can use the product rule and say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

Stephen: That's great. Everyone knows that $\log_2 2 = 1$, but what is $\log_2 3$?

Abe: Oh, I saw this somewhere. Uh, $\log_2 3 = 1.585$. So Mary's idea really works. You say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

$$= 1 + 1.585$$

$$= 2.585$$

$$\log_2 6 = 2.585$$

2. Based on what you know about logarithms, explain why 2.585 is a reasonable value for $\log_2 6$.

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Stephen: Oh, oh! I've got one. I can figure out $\log_2 5$ like this:

$$\log_2(2+3) = \log_2 2 + \log_2 3$$
$$= 1 + 1.585$$
$$= 2.585$$
$$\log_2 5 = 2.585$$

3. Can Stephen and Mary both be correct? Explain who's right, who's wrong (if anyone) and why.

Now you can try applying the *log* rules yourself. Use the values that are given and the ones that you know by definition, like $\log_2 2 = 1$, to figure out each of the following values. Explain what you did in the space below each question.

$\log_2 3 = 1.585$ $\quad$ $\log_2 5 = 2.322$ $\quad$ $\log_2 7 = 2.807$

The three rules, written for any base $b > 1$ are:

Log of a Product Rule:	$\log_b(xy) = \log_b x + \log_b y$
Log of a Quotient Rule:	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
Log of a Power Rule:	$\log_b(x^k) = k\log_b x$

4. $\log_2 9 =$ ____________________

5. $\log_2 10 =$ ____________________

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6. $\log_2 12 =$ ____________________

7. $\log_2 \left(\frac{7}{3}\right) =$ ____________________

8. $\log_2 \left(\frac{30}{7}\right) =$ ____________________

9. $\log_2 486 =$ ____________________

10. Given the work that you have just done, what other values would you need to figure out the value of the base 2 log for any number?

Sometimes thinking about equivalent expressions with logarithms can get tricky. Consider each of the following expressions and decide if they are always true for the numbers in the domain of the logarithmic function, sometimes true, or never true. Explain your answers. If you answer "sometimes true", describe the conditions that must be in place to make the statement true.

11. $\log_4 5 - \log_4 x = \log_4\left(\frac{5}{x}\right)$ ______________________________

12. $\log 3 - \log x - \log x = \log\left(\frac{3}{x^2}\right)$ ______________________________

13. $\log x - \log 5 = \frac{\log x}{\log 5}$ ______________________________

14. $5 \log x = \log x^5$ ______________________________

15. $2 \log x + \log 5 = \log(x^2 + 5)$ ______________________________

16. $\frac{1}{2}\log x = \log\sqrt{x}$ ______________________________

17. $\log(x - 5) = \frac{\log x}{\log 5}$ ______________________________

READY, SET, GO! Name Period Date

READY

Topic: Solving simple exponential and logarithmic equations

You have solved exponential equations before based on the idea that $a^x = a^y$, *if and only if* $x = y$.

You can use the same logic on logarithmic equations. $log_a x = log_a y$, *if and only if* $x = y$

Rewrite each equation so that you set up a one-to-one correspondence between all of the parts. Then solve for x.

Example: *Original equation*	*Rewritten equation:*	*Solution:*
a.) $3^x = 81$	$3^x = 3^4$	$x = 4$
b.) $log_2 x - log_2 5 = 0$	$log_2 x = log_2 5$	$x = 5$

1. $3^{x+4} = 243$

2. $\left(\frac{1}{2}\right)^x = 8$

3. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$

4. $log_2 x - log_2 13 = 0$

5. $log_2(2x - 4) - log_2 8 = 0$

6. $log_2(x + 2) - log_2 9x = 0$

7. $\frac{\log 2x}{\log 14} = 1$

8. $\frac{\log(5x-1)}{\log 29} = 1$

9. $\frac{\log 5^{(x-2)}}{\log 625} = 1$

SET

Topic: Rewriting logs in terms of known logs

Use the given values and the properties of logarithms to find the indicated logarithm.

Do not use a calculator to evaluate the logarithms.

Given: $\log 16 \approx 1.2$
$\log 5 \approx 0.7$
$\log 8 \approx 0.9$

10. Find $\log \frac{5}{8}$

11. Find $\log 25$

12. Find $\log \frac{1}{2}$

13. Find $\log 80$

14. Find $\log \frac{1}{64}$

Given $log_3 2 \approx 0.6$
$log_3 5 \approx 1.5$

15. Find $log_3 16$

16. Find $log_3 108$

17. Find $log_3 \frac{3}{50}$

18. Find $log_3 \frac{8}{15}$

19. Find $log_3 486$

20. Find $log_3 18$

21. Find $log_3 120$

22. Find $log_3 \frac{32}{45}$

GO

Topic: Using the definition of logarithm to solve for x.

Use your calculator and the definition of *log x* (recall the base is 10) to find the value of x.
(Round your answers to 4 decimals.)

23. $\log x = -3$

24. $\log x = 1$

25. $\log x = 0$

26. $\log x = \frac{1}{2}$

27. $\log x = 1.75$

28. $\log x = -2.2$

29. $\log x = 3.67$

30. $\log x = \frac{3}{4}$

31. $\log x = 6$