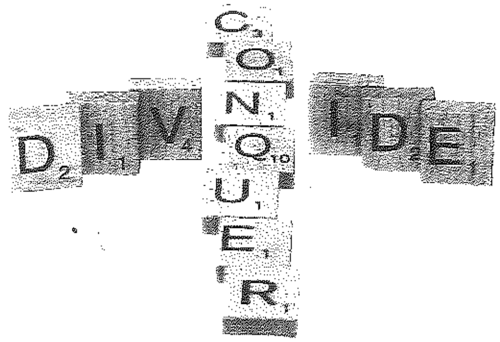
# 3.5 Divide And Conquer

## *A Solidify Understanding Task*





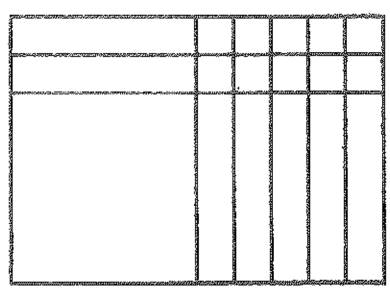

We've seen how numbers and polynomials relate in addition, subtraction, and multiplication. Now we're ready to consider division.

Division, you say? Like, long division? Yup, that's what we're talking about. Hold the judgment! It's actually pretty cool.

As usual, let's start by looking at how the operation works with numbers. Since division is the inverse operation of multiplication, the same models should be useful. The area model that we used with multiplication is also used with division. When we were using area models to factor a quadratic expression, we were actually dividing.

Let's brush up on that a bit.

1. The area model for $x^2 + 7x + 10$ is shown below:



Use the area model to write $x^2 + 7x + 10$ in factored form.

2. We also used number patterns to factor without drawing the area model. Use any strategy to factor the following quadratic polynomials:

| | |
|---|---|
| a) $x^2 + 7x + 12$ | b) $x^2 + 2x - 15$ |



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| | |
|---|---|
| c) $x^2 - 11x + 24$ | d) $x^2 - 5x - 36$ |

Factoring works great for quadratics and a few special cases of other polynomials. Let's look at a more general version of division that is a lot like what we do with numbers. Let's say we want to divide 1452 by 12. If we write the analogous polynomial division problem, it would be: $(x^3 + 4x^2 + 5x + 2) \div (x + 2)$.

Let's use the division process for numbers to create a division process for polynomials. (Don't panic—in many ways it's easier with polynomials than numbers!)

Step 1: Start with writing the problem as long division. The polynomial needs to have the terms written in descending order. If there are any missing powers, it's easier if you leave a little space for them.

$$12\overline{)1452} \qquad\qquad x+2\overline{)x^3+4x^2+5x+2}$$

Step 2: Determine what you could multiply the divisor by to get the first term of the dividend.

$$\begin{array}{r} 1\phantom{452} \\ 12\overline{)1452} \end{array} \qquad\qquad \begin{array}{r} x^2\phantom{+4x^2+5x+2} \\ x+2\overline{)x^3+4x^2+5x+2} \end{array}$$

Step 3: Multiply and put the result below the dividend.

$$\begin{array}{r} 1\phantom{452} \\ 12\overline{)1452} \\ \underline{-1200} \end{array} \qquad\qquad \begin{array}{r} x^2\phantom{+4x^2+5x+2} \\ x+2\overline{)x^3+4x^2+5x+2} \\ -(\underline{x^3+2x^2})\phantom{+5x+2} \end{array}$$

Step 4: Subtract. (It helps to keep the signs straight if you change the sign on each term and add on the polynomial.)

$$\begin{array}{r} 1 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \end{array}$$

$$\begin{array}{r} x^2 \qquad\qquad\quad \\ x+2\overline{)x^3+4x^2+5x+2} \\ +\underline{(-x^3-2x^2 \qquad\quad )} \\ 2x^2+5x+2 \end{array}$$

Step 5: Repeat the process with the number or expression that remains in the dividend.

$$\begin{array}{r} 12 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \end{array}$$

$$\begin{array}{r} x^2+2x \qquad\quad \\ x+2\overline{)x^3+4x^2+5x+2} \\ +\underline{(-x^3-2x^2 \qquad\quad )} \\ 2x^2+5x+2 \\ -\underline{(2x^2+4x \quad )} \\ x+2 \end{array}$$

Step 6: Keep going until the number or expression that remains is smaller than the divisor.

$$\begin{array}{r} 121 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \\ -\underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} x^2+2x+1 \\ x+2\overline{)x^3+4x^2+5x+2} \\ +\underline{(-x^3-2x^2 \qquad\quad )} \\ 2x^2+5x+2 \\ -\underline{(2x^2+4x \quad )} \\ x+2 \\ -\underline{(x+2)} \\ 0 \end{array}$$

In this case, 121 divided by 12 leaves no remainder, so we would say that 12 is a factor of 121. Similarly, since $(x^3+4x^2+5x+2)$ divided by $(x+2)$ leaves no remainder, we would say that $(x+2)$ is a factor of $(x^3+4x^2+5x+2)$.

Polynomial division doesn't always match up perfectly to an analogous whole number problem, but the process is always the same. Let's try it.

3. Use long division to determine if $(x-1)$ a factor of $(x^3-3x^2-13x+15)$. Don't worry: the steps for the division process are below:

a) Write the problem as long division.
b) What do you have to multiply $x$ by to get $x^3$? Write your answer above the bar.
c) Multiply your answer from step b by $(x-1)$ and write your answer below the dividend.
d) Subtract. Be careful to subtract each term. (You might want to change the signs and add.)
e) Repeat steps a-d until the expression that remains is less than $(x-1)$.

We hope you survived the division process. Is $(x-1)$ a factor of $(x^3-3x^2-13x+15)$? ______

4. Try it again. Use long division to determine if $(2x+3)$ is a factor of $2x^3+7x^2+2x+9$. No hints this time. You can do it!

When dividing numbers, there are several ways to deal with the remainder. Sometimes, we just write it as the remainder, like this:

$$3\overline{)\,25}\quad 8r.1$$

because $3(8)+1=25$

You may remember also writing the remainder as a fraction like this:

$$3\overline{)\,25\,}\quad 8\tfrac{1}{3}$$ because $3\left(8\frac{1}{3}\right) = 25$

We do the same things with polynomials.

Maybe you found that $(2x^3 + 7x^2 + 2x + 9) \div (2x + 3) = (x^2 + 2x - 2)\ r.\ 15$. (We sure hope so.) You can use it to write two multiplication statements:

$$(2x + 3)(x^2 + 2x - 2) + 15 = (2x^3 + 7x^2 + 2x + 9)$$

and

$$(2x + 3)\left(x^2 + 2x - 2 + \frac{15}{2x + 3}\right) = (2x^3 + 7x^2 + 2x + 9)$$

5. Divide each of the following polynomials. Write the two multiplication statements that go with your answers if there is a remainder. Write only one multiplication statement if the divisor is a factor. Use graphing technology to check your work and make the necessary corrections.

| | | |
|---|---|---|
| | a) $(x^3 + 6x^2 + 13x + 12) \div (x + 3)$ | b) $(x^3 - 4x^2 + 2x + 5) \div (x - 2)$ |
| Multiplication statements: | | |
| | | |



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| | | |
|---|---|---|
| | c) $(6x^3 - 11x^2 - 4x + 5) \div (2x - 1)$ | d) $(x^4 - 23x^3 + 49x + 4) \div (x^2 + x + 2)$ |
| Multiplication statements: | | |
| | | |

3.5

**READY, SET, GO!** Name Period Date

## READY

Topic: Solving linear equations

**Solve for x.**

1. $5x + 13 = 48$
2. $\frac{1}{3}x - 8 = 0$
3. $-4 - 9x = 0$
4. $x^2 - 16 = 0$
5. $x^2 + 4x + 3 = 0$
6. $x^2 - 5x + 6 = 0$
7. $(x + 8)(x + 11) = 0$
8. $(x - 5)(x - 7) = 0$
9. $(3x - 18)(5x - 10) = 0$

## SET

Topic: Dividing polynomials

**Divide each of the following polynomials. Write only one multiplication statement if the divisor is a factor. Write the two multiplication statements that go with your answers if there is a remainder.**

10. $(x+1)\overline{)x^3 - 3x^2 + 6x + 11}$

Multiplication statement(s)

11. $(x-5)\overline{)x^3 - 9x^2 + 23x - 15}$

Multiplication statement(s)





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12. $(2x-1)\overline{)2x^3+15x^2-34x+13}$

Multiplication statement(s)

13. $(x+4)\overline{)x^3+13x^2+26x-25}$

Multiplication statement(s)

14. $(x+7)\overline{)x^3-8x^2-111x+10}$

Multiplication statement(s)

15. $(3x-4)\overline{)3x^3+23x^2+6x-28}$

Multiplication statement(s)

GO

Topic: Describing the features of a variety of functions

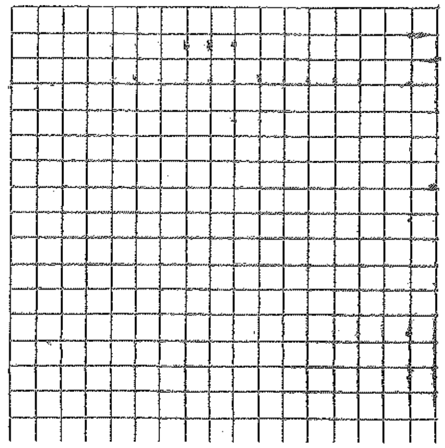
**Graph the following functions. Then identify the key features of the functions. Include domain, range, intervals where the function is increasing/decreasing, intercepts, maximum/minimum, and end behavior.**

16. $f(x) = x^2 - 9$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

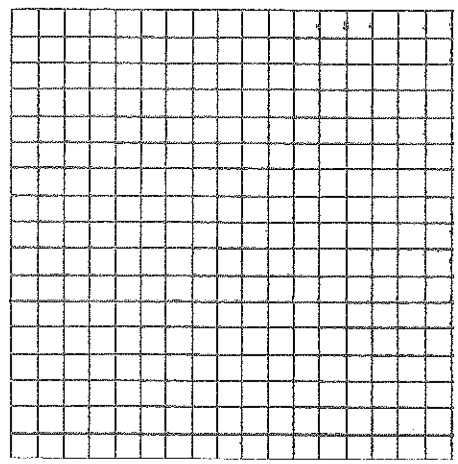


17. $f(n-1) = f(n) + 3;\ f(1) = 4$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

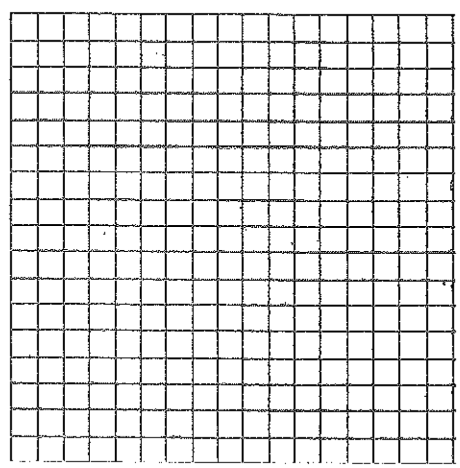


18. $f(x) = \sqrt{x-3} + 1$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

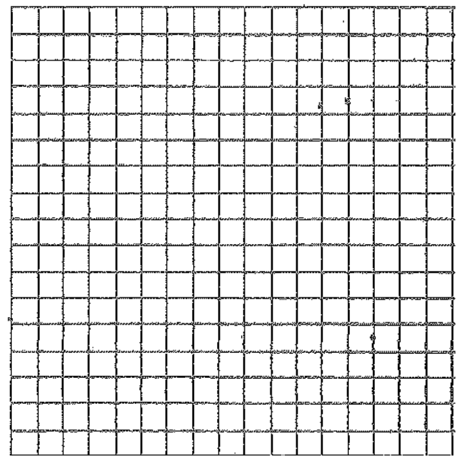


19. $f(x) = log_2 x - 1$

domain: range:

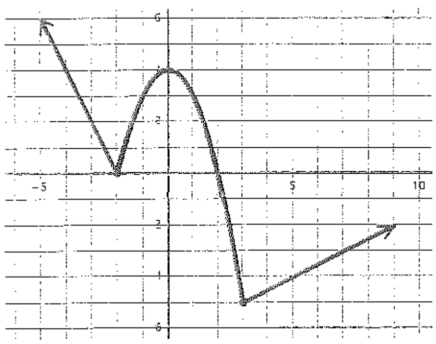
increasing: decreasing:

y-intercept: x-intercept(s):



**Identify the key features of the graphed functions.**

20.

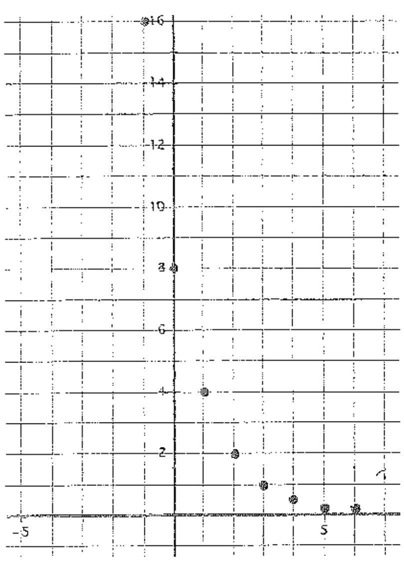



domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

21.




domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):