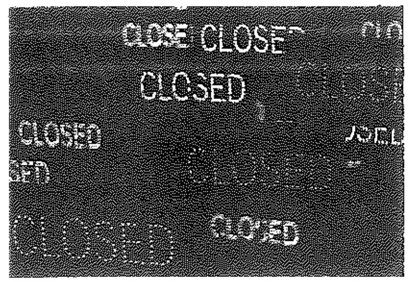
3.6 Sorry, We're Closed

A Practice Understanding Task



Now that we have compared operations on polynomials with operations on whole numbers it's time to generalize about the results. Before we go too far, we need a technical definition of a polynomial function. Here it is:

A polynomial function has the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots a_1, a_0$ **are real numbers and** n **is a nonnegative integer. In other words, a polynomial is the sum of one or more monomials with real coefficients and nonnegative integer exponents. The degree of the polynomial function is the highest value for n where** a_n **is not equal to 0.**

1. The following examples and non-examples will help you to see the important implications of the definition of a polynomial function. For each pair, determine what is different between the example of a polynomial and the non-example that is not a polynomial.

These are polynomials:	These are not polynomials:
a) $f(x) = x^3$	b) $g(x) = 3^x$
How are a and b different?	
c) $f(x) = 2x^2 + 5x - 12$	d) $g(x) = \frac{2x^2}{x^2-3x+2}$
How are c and d different?	
e) $f(x) = -x^3 + 3x^2 - 2x - 7$	f) $g(x) = x^3 + 3x^2 - 2x + 10x^{-1} - 7$
How are e and f different?	
h) $f(x) = \frac{1}{2}x$	i) $g(x) = \frac{1}{2x}$
How are h and i different?	
j) $f(x) = x^2$	k) $g(x) = x^{\frac{1}{2}}$
How are j and k different?	

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2. Based on the definition and the examples above, how can you tell if a function is a polynomial function?

Maybe you have noticed in the past that when you add two even numbers, the answer you get is always an even number. Mathematically, we say that the set of even numbers is **closed under** addition. Mathematicians are interested in results like this because it helps us to understand how numbers or functions of a particular type behave with the various operations.

3. You can try it yourself: Is the set of odd numbers closed under multiplication? In other words, if you multiply two odd numbers together will you get an odd number? Explain.

If you find any two odd numbers that have an even product, then you would say that odd numbers are not closed under multiplication. Even if you have a number of examples that support the claim, if you can find one **counterexample** that contradicts the claim, then the claim is false.

Consider the following claims and determine whether they are true or false. If a claim is true, give a reason with at least two examples that illustrate the claim. Your examples can include any representation you choose. If the claim is false, give a reason with one counterexample that proves the claim to be false.

4. The set of whole numbers is closed under addition.

5. The sum of a quadratic function and a linear function is a cubic function.

6. The sum of a linear function and an exponential function is a polynomial.

7. The set of polynomials is closed under addition.

8. The set of whole numbers is closed under subtraction.

9. The set of integers is closed under subtraction.

10. A quadratic function subtracted from a cubic function is a cubic function.

11. A linear function subtracted from a linear function is a polynomial function.

12. A cubic function subtracted from a cubic function is a cubic function.

13. The set of polynomial functions is closed under subtraction.

14. The product of two linear functions is a quadratic function.

15. The set of integers is closed under multiplication.

16. The set of polynomials is closed under multiplication.

17. The set of integers is closed under division.

18. A cubic function divided by a linear function is a quadratic function.

19. The set of polynomial functions is closed under division.

20. Write two claims of your own about polynomials and use examples to demonstrate that they are true.

Claim #1:

Claim #2:

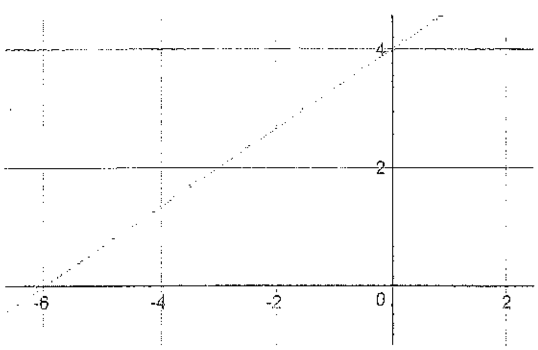
READY, SET, GO! Name Period Date

READY

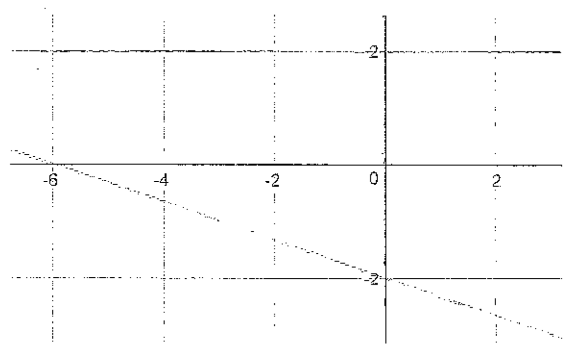
Topic: Connecting the zeros of a function to the solution of the equation

When we solve equations, we often set the equation equal to zero and then find the value of x. Another way to say this is "find when $f(x) = 0$." That's why we call solutions to equations the zeros of an equation. Find the zeros for the given equations. Then mark the solution(s) as a point on the graph of the equation.

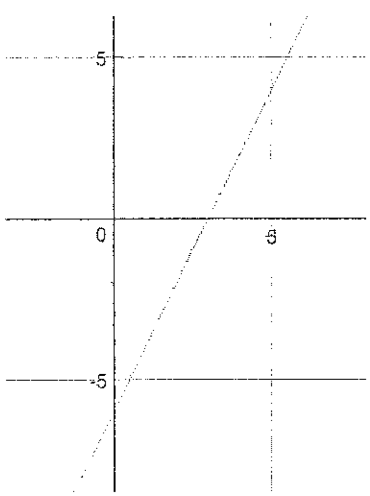
1. $f(x) = \frac{2}{3}x + 4$



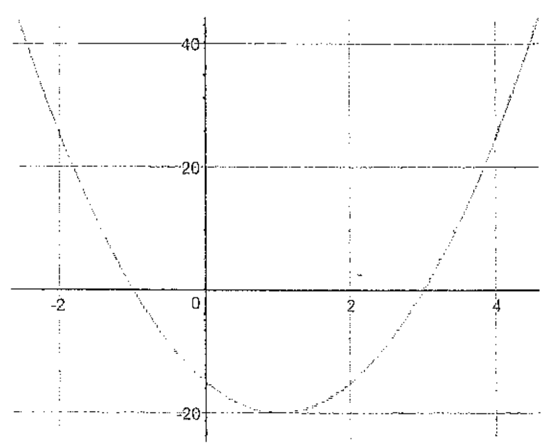
2. $g(x) = -\frac{1}{3}x - 2$



3. $h(x) = 2x - 6$



4. $p(x) = 5x^2 - 10x - 15$



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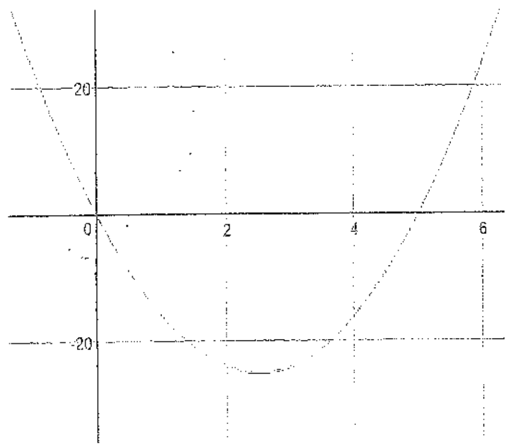
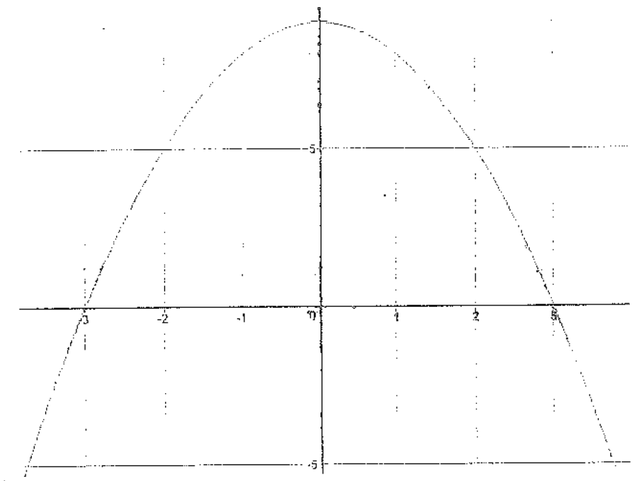
5. $q(x) = 4x^2 - 20x$



6. $d(x) = -x^2 + 9$



SET

Topic: Exploring closed mathematical number sets

Identify the following statements as *sometimes* true, *always* true, or *never* true. If your answer is *sometimes* true, give an example of when it's true and an example of when it's not true. If it's *never* true, give a counter-example.

7. The product of a whole number and a whole number is an integer.

8. The quotient of a whole number divided by a whole number is a whole number.

9. The set of integers is closed under division.

10. The difference of a linear function and a linear function is an integer.

11. The difference of a linear function and a quadratic function is a linear function.

12. The product of a linear function and a linear function is a quadratic function.

13. The sum of a quadratic function and a quadratic function is a polynomial function.

14. The product of a linear function and a quadratic function is a cubic function.

15. The product of three linear functions is a cubic function.

16. The set of polynomial functions is **closed** under addition.

GO

Topic: Identifying conjugate pairs

A **conjugate pair** is simply a pair of binomials that have the same numbers but differ by having opposite signs between them. For example $(a + b)$ and $(a - b)$ are conjugate pairs. You've probably noticed them when you've factored a quadratic expression that is the difference of two squares. **Example:** $x^2 - 25 = (x+5)(x-5)$. The two factors $(x+5)(x-5)$ are conjugate pairs.

The quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ can generate both solutions to a quadratic equation because of the $\pm$ located in the numerator of the formula. When the $\sqrt{b^2 - 4ac}$ part of the formula generates an irrational number (e.g. $\sqrt{2}$) or an imaginary number (e.g. $2i$), the formula produces a pair of numbers that are conjugates. This is important because this type of solution to a quadratic always comes in pairs. **Example:** The conjugate of $(3 + \sqrt{2})$ *is* $(3 - \sqrt{2})$. The conjugate of $(-2i)$ is $(+2i)$. Think of it as $(0 - 2i)$ *and* $(0 + 2i)$. Change only the sign between the two numbers.

Write the conjugate of the given value.

17. $(8+\sqrt{5})$ 18. $(11+4i)$ 19. $9i$ 20. $-5\sqrt{7}$

21. $(2-13i)$ 22. $(-1-2i)$ 23. $(-3+5\sqrt{2})$ 24. $-4i$