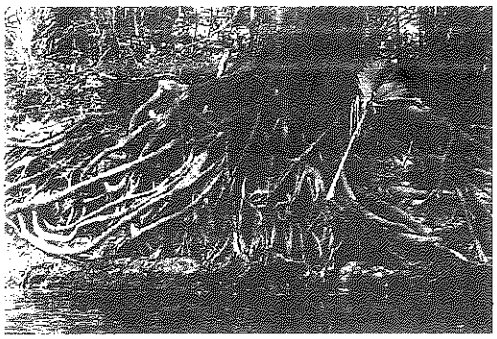
3.7 Building Strong Roots

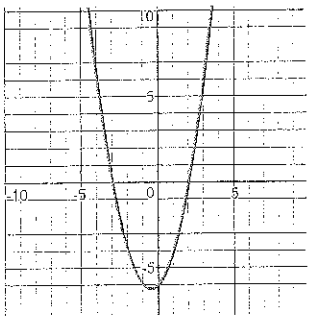
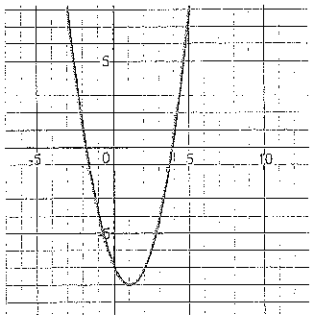
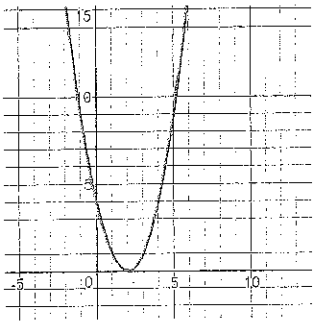
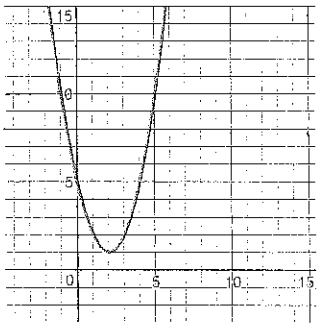
A Solidify Understanding Task



When working with quadratic functions, we learned the Fundamental Theorem of Algebra:
An n^{th} degree polynomial function has n roots.
In this task, we will be exploring this idea further with other polynomial functions.

First, let's brush up on what we learned about quadratics. The equations and graphs of four different quadratic equations are given below. Find the roots for each and identify whether the roots are real or imaginary.

1.

a) $f(x) = x^2 + x - 6$ 	b) $g(x) = x^2 - 2x - 7$ 
Roots:	Roots:
Type of roots:	Type of roots:
c) $h(x) = x^2 - 4x + 4$ 	d) $k(x) = x^2 - 4x + 5$ 
Roots:	Roots:
Type of roots:	Type of roots:

2. Did all of the quadratic functions have 2 roots, as predicted by the Fundamental Theorem of Algebra? Explain.

3. It's always important to keep what you've previously learned in your mathematical bag of tricks so that you can pull it out when you need it. What strategies did you use to find the roots of the quadratic equations?

4. Using your work from problem 1, write each of the quadratic equations in factored form. When you finish, check your answers by graphing, when possible, and make any corrections necessary.

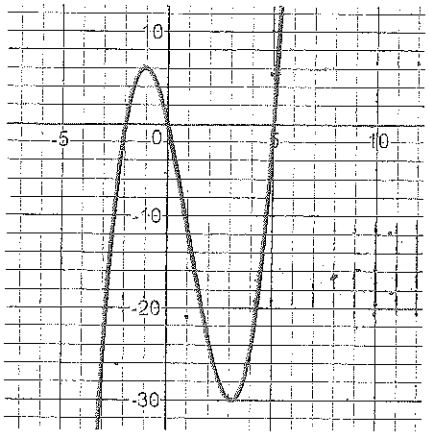
a) $f(x) = x^2 + x - 6$	b) $g(x) = x^2 - 2x - 7$
Factored form:	Factored form:
c) $h(x) = x^2 - 4x + 4$	d) $k(x) = x^2 - 4x + 5$
Factored form:	Factored form:

5. Based on your work in problem 1, would you say that roots are the same as x-intercepts? Explain.

6. Based on your work in problem 4, what is the relationship between roots and factors?

Now let's take a closer look at cubic functions. We've worked with transformations of $f(x) = x^3$, but what we've seen so far is just the tip of the iceberg. For instance, consider:

$g(x) = x^3 - 3x^2 - 10x$



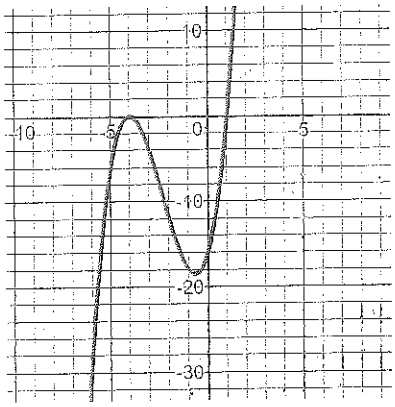
7. Use the graph to find the roots of the cubic function. Use the equation to verify that you are correct. Show how you have verified each root.

8. Write $g(x)$ in factored form. Verify that the factored form is equivalent to the standard form.

9. Are the results you found in #7 consistent with the Fundamental Theorem of Algebra? Explain.

Here's another example of a cubic function.

$f(x) = x^3 + 7x^2 + 8x - 16$

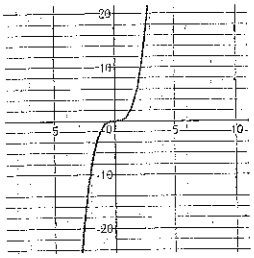


10. Use the graph to find the roots of the cubic function.

11. Write $f(x)$ in factored form. Verify that the factored form is equivalent to the standard form. Make any corrections needed.

12. Are the results you found in #10 consistent with the Fundamental Theorem of Algebra? Explain.

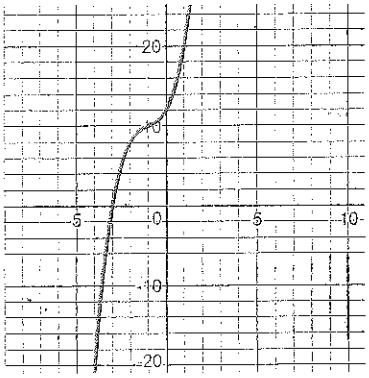
13. We've seen the most basic cubic polynomial function, $h(x) = x^3$ and we know its graph looks like this:



Explain how $h(x) = x^3$ is consistent with the Fundamental Theorem of Algebra.

14. Here is one more cubic polynomial function for your consideration. You will notice that it is given to you in factored form. Use the equation and the graph to find the roots of $p(x)$.

$$p(x) = (x + 3)(x^2 + 4)$$



15. Use the equation to verify each root. Show your work below.

16. Are the results you found in #14 consistent with the Fundamental Theorem of Algebra? Explain.

17. Explain how to find the factored form of a polynomial, given the roots.

18. Explain how to find the roots of a polynomial, given the factored form.

3.7

READY, SET, GO! Name Period Date

READY

Topic: Practicing long division on polynomials

Divide using long division. (These problems have no remainders. If you get one, try again.)

1. $(x+3)\overline{)5x^3+2x^2-45x-18}$

2. $(x-6)\overline{)x^3-x^2-44x+84}$

3. $(x-5)\overline{)3x^3-15x^2+12x-60}$

4. $(x+2)\overline{)x^4+6x^3+7x^2-6x-8}$

SET

Topic: Applying the Fundamental Theorem of Algebra

Predict the number of roots for each of the given polynomial equations. (Remember that the Fundamental Theorem of Algebra states: An n^{th} degree polynomial function has n roots.)

5. $a(x) = x^2 + 3x - 10$

6. $b(x) = x^3 + x^2 - 9x - 9$

7. $c(x) = -2x - 4$

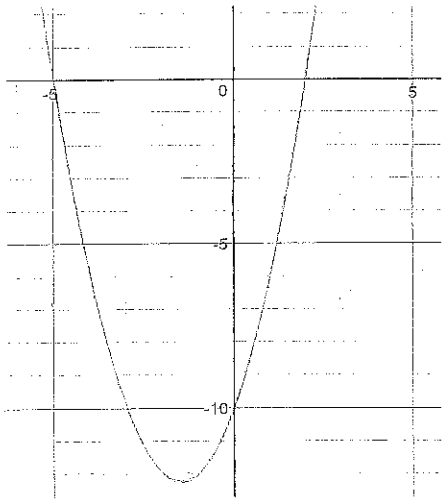
8. $d(x) = x^4 - x^3 - 4x^2 + 4x$

9. $f(x) = -x^2 + 6x - 9$

10. $g(x) = x^6 - 5x^4 + 4x^2$

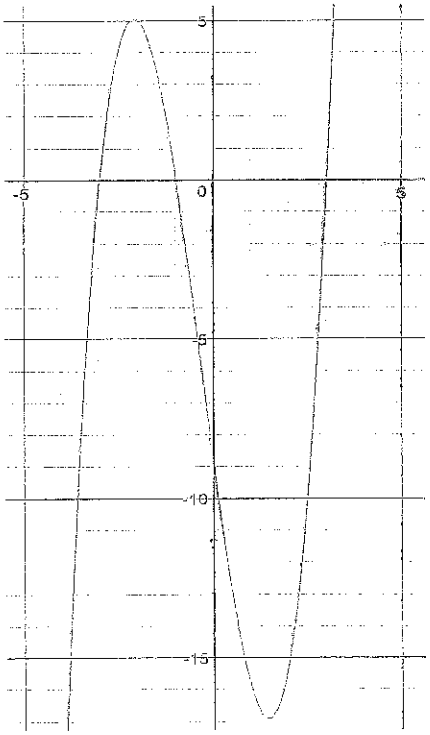
Below are the graphs of the polynomials from the previous page. Check your predictions. Then use the graph to help you write the polynomial in factored form.

11. $a(x) = x^2 + 3x - 10$



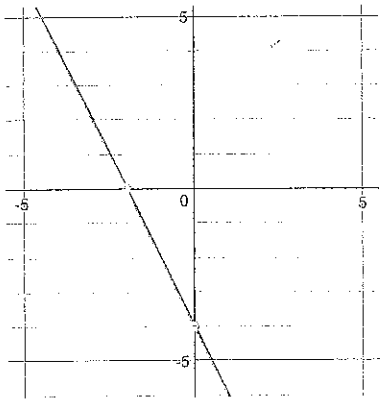
Factored form:

12. $b(x) = x^3 + x^2 - 9x - 9$



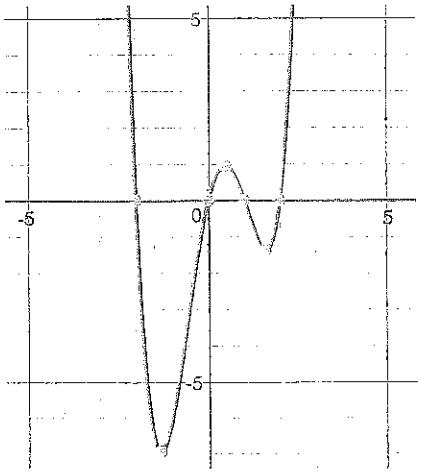
Factored form:

13. $c(x) = -2x - 4$



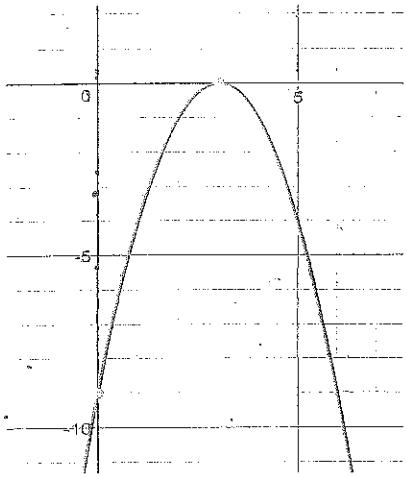
Factored form:

14. $d(x) = x^4 - x^3 - 4x^2 + 4x$



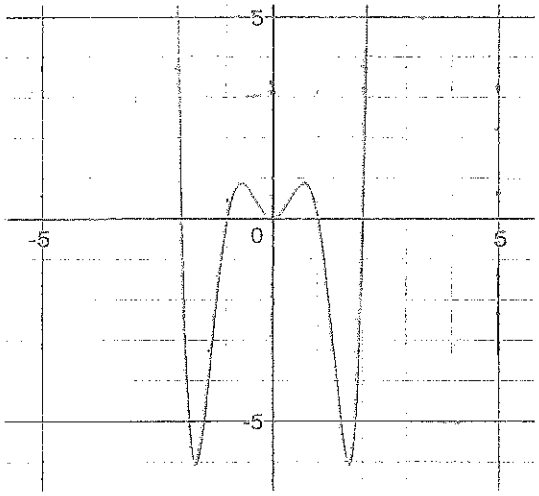
Factored form:

15. $f(x) = -x^2 + 6x - 9$



Factored form:

16. $g(x) = x^6 - 5x^4 + 4x^2$



Factored form:

17. The graphs of #15 and #16 don't seem to follow the Fundamental Theorem of Algebra, but there is something similar about each of the graphs. Explain what is happening at the point (3, 0) in #15 and at the point (0,0) in #16.

GO

Topic: Solving quadratic equations

Find the zeros for each equation using the quadratic formula.

18. $f(x) = x^2 + 20x + 51$

19. $f(x) = x^2 + 10x + 25$

20. $f(x) = 3x^2 + 12x$

21. $f(x) = x^2 - 11$

22. $f(x) = x^2 + x - 1$

23. $f(x) = x^2 + 2x + 3$