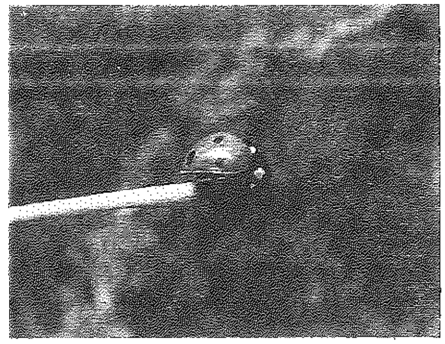
3.9 Is This The End?

A Solidify Understanding Task



CC BY Caitlin H
https://flic.kr/p/6S4Xqn

In previous mathematics courses, you have compared and analyzed growth rates of polynomial (mostly linear and quadratic) and exponential functions. In this task, we are going to analyze rates of change and end behavior by comparing various expressions.

Part I: Seeing patterns in **end behavior**

1. In as many ways as possible, compare and contrast linear, quadratic, cubic, and exponential functions.

2. Using the graph provided, write the following functions vertically, from **greatest to least** for $x = 0$. Put the function with the greatest value on top and the function with the smallest value on the bottom. Put functions with the same values at the same level. An example, $l(x) = x^7$, has been placed on the graph to get you started.

$f(x) = 2^x$	$p(x) = x^3 + x^2 - 4$	$g(x) = x^2 - 20$
$h(x) = x^5 - 4x^2 + 1$	$k(x) = x + 30$	$m(x) = x^4 - 1$
$r(x) = x^5$	$n(x) = \left(\frac{1}{2}\right)^x$	$q(x) = x^6$

3. What determines the value of a polynomial function at $x = 0$? Is this true for other types of functions?

4. Write the same expressions on the graph in order from **greatest to least** when x represents a very large number (this number is so large, so we say that it is approaching positive infinity). If the value of the function is positive, put the function in quadrant 1. If the value of the function is negative, put the function in quadrant IV. An example has been placed for you.

5. What determines the end behavior of a polynomial function for very large values of x?

6. Write the same functions in order from greatest to least when x represents a number that is approaching negative infinity. If the value of the function is positive, place it in Quadrant II, if the value of the function is negative, place it in Quadrant III. An example is shown on the graph.

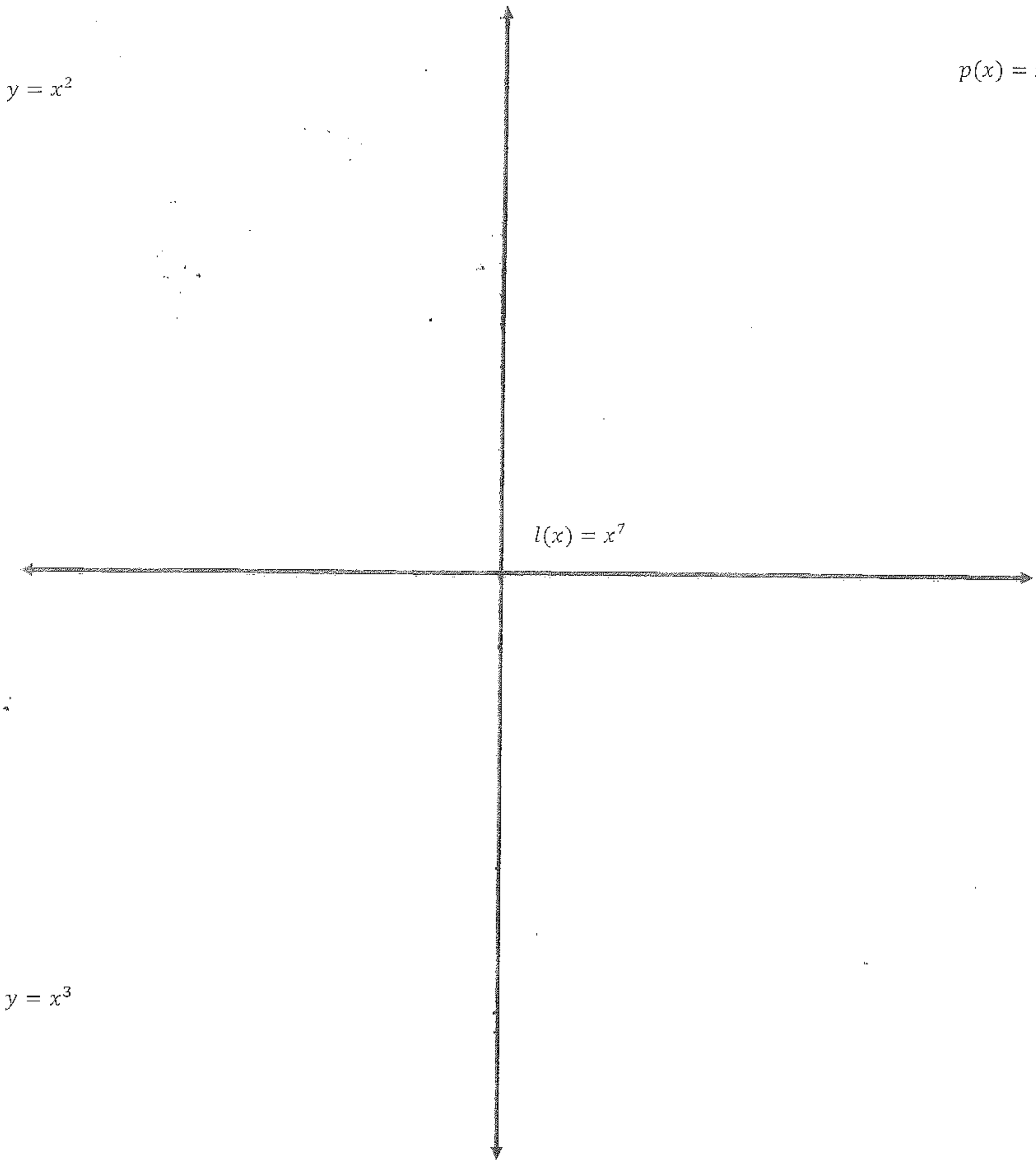
7. What patterns do you see in the polynomial functions for x values approaching negative infinity? What patterns do you see for exponential functions? Use graphing technology to test these patterns with a few more examples of your choice.

8. How would the end behavior of the polynomial functions change if the lead terms were changed from positive to negative?

$x \to -\infty$ | $x = 0$ | $x \to \infty$

$y = x^2$

$p(x) = x^7$



$y = x^3$

Part II: Using end behavior patterns

For each situation:

- Determine the function type. If it is a polynomial, state the degree of the polynomial and whether it is an even degree polynomial or an odd degree polynomial.
- Describe the end behavior based on your knowledge of the function. Use the format:
  As $x \to -\infty, f(x) \to$ ____ *and* as $x \to \infty\ f(x) \to$ ____

1. $f(x) = 3 + 2x$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

2. $f(x) = x^4 - 16$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

3. $f(x) = 3^x$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

4. $f(x) = x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

5. $f(x) = -2x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

6. $f(x) = log_2 x$

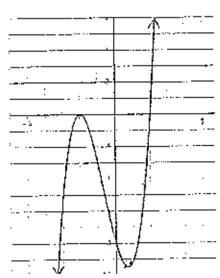
   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ ____

   End behavior: As $x \to \infty, f(x) \to$ ____

Use the graphs below to describe the end behavior of each function by completing the statements.

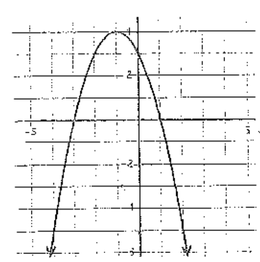
7.



End behavior: As $x \to -\infty, f(x) \to$ ____

End behavior: As $x \to \infty, f(x) \to$ ____

8.



End behavior: As $x \to -\infty, f(x) \to$ ____

End behavior: As $x \to \infty, f(x) \to$ ____

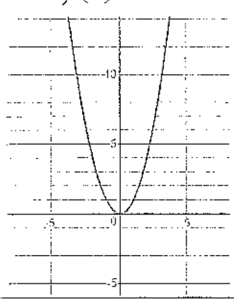
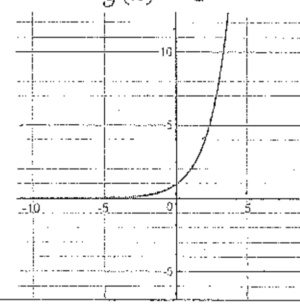
mathematics vision project

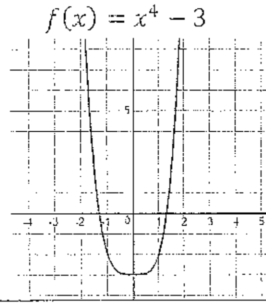
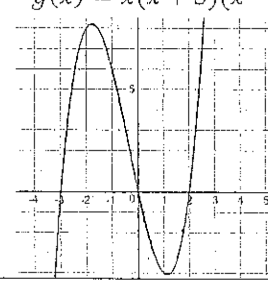
9. How does the end behavior for quadratic functions connect with the number and type of roots for these functions? How does the end behavior for cubic functions connect with the number and type of roots for cubic functions?

Part III: Even and Odd Functions

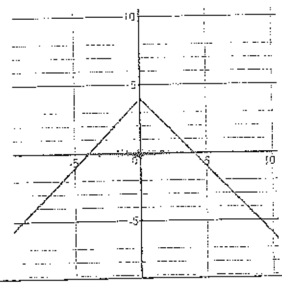
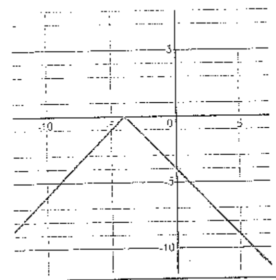
Some functions that are not polynomials may be categorized as even functions or odd functions. When mathematicians say that a function is an even function, they mean something very specific.

1. Let's see if you can figure out what the definition of an even function is with these examples:

Even function: $f(x) = x^2$	Not an even function: $g(x) = 2^x$
	
Differences:	

Even function: $f(x) = x^4 - 3$	Not an even function: $g(x) = x(x+3)(x-2)$
	
Differences:	

mathematics vision project

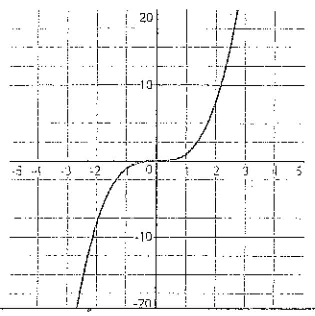
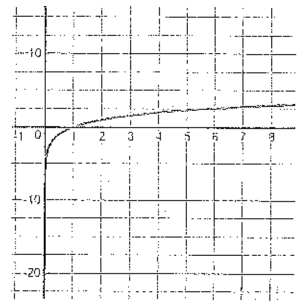
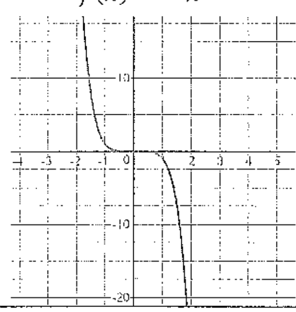
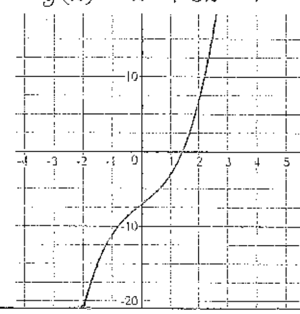
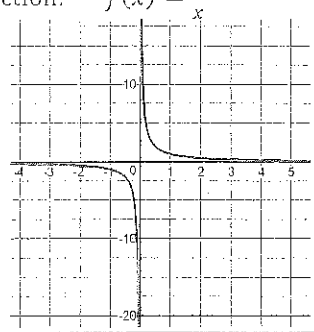
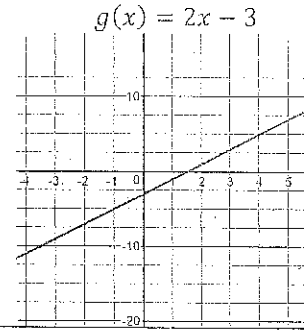
Even function: $f(x) = -\lvert x \rvert + 4$ 	Not an even function: $g(x) = -\lvert x + 4 \rvert$ 
Differences:	
Even function: $f(2) = 5 \text{ and } f(-2) = 5$	Not an even function: $g(2) = 3 \text{ and } g(-2) = 5$
Differences:	

2. What do you observe about the characteristics of an even function?

3. The algebraic definition of an even function is:
   $f(x)$ **is an even function if and only if** $f(x) = f(-x)$ **for all values of** x **in the domain of** f.
   What are the implications of the definition for the graph of an even function?

4. Are all even-degree polynomials even functions? Use examples to explain your answer.

5. Let's try the same approach to figure out a definition for odd functions.

Odd function: $f(x) = x^3$ 	Not an odd function: $g(x) = \log_2 x$ 
Differences:	
Odd function: $f(x) = -x^5$ 	Not an odd function: $g(x) = x^3 + 3x - 7$ 
Differences:	
Odd function: $f(x) = \frac{1}{x}$ 	Not an odd function: $g(x) = 2x - 3$ 
Differences:	
Odd function: $f(2) = 3 \ and\ f(-2) = -3$	Not an odd function: $g(2) = 3 \ and\ g(-2) = 5$
Differences:	

6. What do you observe about the characteristics of an odd function?

7. The algebraic definition of an odd function is:

   $f(x)$ is an odd function if and only if $f(-x) = -f(x)$ for all values of x in the domain of f.

Explain how each of the examples of odd functions above meet this definition.

8. How can you tell if an odd-degree polynomial is an odd function?

9. Are all functions either odd or even?

READY, SET, GO! Name Period Date

READY

Topic: Recognizing special products

Multiply.

1. $(x+5)(x+5)$ 2. $(x-3)(x-3)$ 3. $(a+b)(a+b)$

4. In problems 1 – 3 the answers are called **perfect square trinomials.** What about these answers makes them be a **perfect square trinomial?**

5. $(x+8)(x-8)$ 6. $(x+\sqrt{3})(x-\sqrt{3})$ 7. $(x+b)(x-b)$

8. The products in problems 5 – 7 end up being binomials, and they are called the **difference of two squares.** What about these answers makes them be the **difference of two squares?**

   Why don't they have a middle term like the problems in 1 – 3?

9. $(x-3)(x^2+3x+9)$ 10. $(x+10)(x^2-10x+100)$ 11. $(a+b)(a^2-ab+b^2)$

12. The work in problems 9 – 11 makes them feel like the answers are going to have a lot of terms. What happens in the work of the problem that makes the answers be binomials?

    These answers are called the **difference of two cubes** (#9) and the **sum of two cubes** (#10 and #11.) What about these answers makes them be the **sum or difference of two cubes?**

SET

Topic: Determining values of polynomials at zero and at $\pm\infty$. (End behavior)

State the y-intercept, the degree, and the end behavior for each of the given polynomials.

13. $f(x) = x^5 + 7x^4 - 9x^3 + x^2 - 13x + 8$

y- intercept:

Degree:

End behavior:

As $x \to -\infty$, $f(x) \to$ ______

As $x \to +\infty$, $f(x) \to$ ______

14. $g(x) = 3x^4 + x^3 + 5x^2 - x - 15$

y- intercept:

Degree:

End behavior:

As $x \to -\infty$, $g(x) \to$ ______

As $x \to +\infty$, $g(x) \to$ ______

15. $h(x) = -7x^9 + x^2$

y- intercept:

Degree:

End behavior:

As $x \to -\infty$, $h(x) \to$ ______

As $x \to +\infty$, $h(x) \to$ ______

16. $p(x) = 5x^2 - 18x + 4$

y- intercept:

Degree:

End behavior:

As $x \to -\infty$, $p(x) \to$ ______

As $x \to +\infty$, $p(x) \to$ ______

17. $q(x) = x^3 - 94x^2 - x - 20$

y- intercept:

Degree:

End behavior:

As $x \to -\infty$, $q(x) \to$ ______

As $x \to +\infty$, $q(x) \to$ ______

18. $y = -4x + 12$

y- intercept:

Degree:

End behavior:

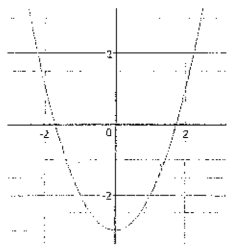
As $x \to -\infty$, $y \to$ ______

As $x \to +\infty$, $y \to$ ______

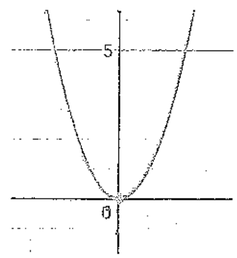
Topic: Identifying even and odd functions

19. Identify each function as even, odd, or neither.

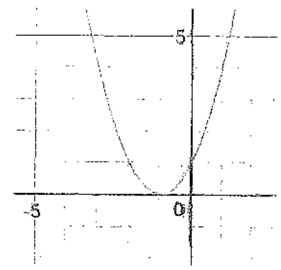
a) $f(x) = x^2 - 3$



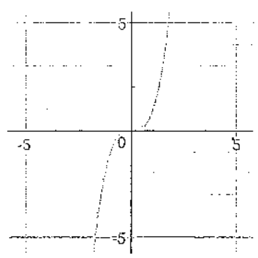
b) $f(x) = x^2$



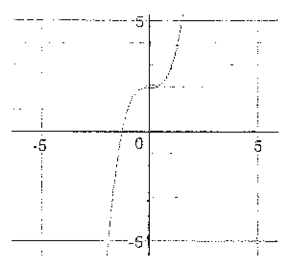
c) $f(x) = (x + 1)^2$



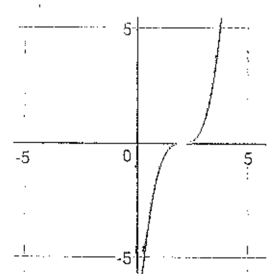
d) $f(x) = x^3$



e) $f(x) = x^3 + 2$



f) $f(x) = (x - 2)^3$



GO

Topic: Factoring special products

Fill in the blanks on the sentences below.

20. The expression $a^2 + 2ab + b^2$ is called a **perfect square trinomial**. I can recognize it because the first and last terms will always be perfect ____________________.

The middle term will be 2 times the ________________ and ________________.

There will always be a __________ sign before the last term.

It factors as (__________)(__________).

21. The expression $a^2 - b^2$ is called the **difference of 2 squares**. I can recognize it because it's a binomial and the first and last terms are perfect ______________________.
The sign between the first term and the last term is always a ______________.
It factors as (__________)(__________).

22. The expression $a^3 + b^3$ is called the **sum of 2 cubes**. I can recognize it because it's a binomial and the first and last terms are __________________. The expression $a^3 + b^3$ factors into a binomial and a trinomial. I can remember it as a *short* (____) and a *long* (__________).
The sign between the terms in the binomial is the ____________ as the sign in the expression. The first sign in the trinomial is the ______________ of the sign in the binomial. That's why all of the middle terms cancel when multiplying.
The last sign in the trinomial is always _______.
It factors as (__________)(__________________).

Factor using what you know about special products.

23. $25x^2 + 30 + 9$

24. $x^2 - 16$

25. $x^3 + 27$

26. $49x^2 - 36$

27. $x^3 - 1$

28. $64x^2 - 240 + 225$