4.4 Are You Rational?

A Solidify Understanding Task

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Back in Module 3 when we were working with polynomials, it was useful to draw connections between polynomials and integers. In this task, we will use connections between rational numbers and rational functions to help us to think about operations on rational functions.

1. In your own words, define **rational number.**

Circle the numbers below that are rational and refine your definition, if needed.

$3 \quad -5 \quad \frac{2}{3} \quad \frac{20}{3} \quad 14 \quad 2.7 \quad \sqrt{5} \quad 2^3 \quad 3^{-3} \quad log_2 9 \quad \frac{7}{0}$

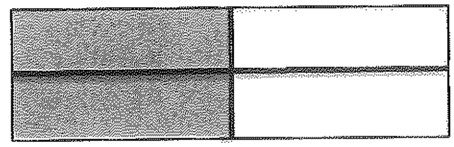
2. The formal definition of a **rational function** is as follows:

A function $f(x)$ is called a rational function if and only if it can be written in the form $f(x) = \frac{P(x)}{Q(x)}$ where P *and* Q are polynomials in x and Q is not the zero polynomial.

Interpret this definition in your own words and then write three examples of rational functions.

3. How are rational numbers and rational functions similar? Different?

Now we are going to use what we know about rational numbers to perform operations on rational expressions. The first thing we often need to do is to simplify or "reduce" a rational number or expression. The numbers and expressions are not really being reduced because the value isn't actually changing. For instance, 2/4 can be simplified to ½, but as the diagram shows, these are just two different ways of expressing the same amount.



Let's try using what we know about simplifying rational numbers to simplify rational expressions. Fill in any missing parts in the fractions below.

Given:	$\frac{24}{30}$	4. $\frac{x^2-x-6}{x^2-4}$	5. $\frac{x^2+8x+15}{x^2+9x+18}$
Look for common factors:	$\frac{2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 5}$	$\frac{}{(x+2)(x-2)}$	
Divide numerator and denominator by the same factor(s):	$\frac{\cancel{2}\cdot 2\cdot 2\cdot \cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 5}$		
Write the simplified form:	$\frac{4}{5}$	$\frac{x-3}{x-2}$	$\frac{x+5}{x+6}$

6. Why does dividing the numerator and denominator by the same factor keep the value of the expression the same?

7. If you were given the expression $\frac{x}{x^2-1}$, would it be acceptable to reduce it like this:

$$\frac{\cancel{x}}{\cancel{x^2}-1}=\frac{1}{x-1}$$

Explain your answer.

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In *4.3 Rational Thinking,* we learned to predict vertical and horizontal asymptotes, and to find intercepts for graphing rational functions.

8. Given $f(x) = \frac{x^2-x-6}{x^2-4}$, predict the vertical and horizontal asymptotes and find the intercepts.

9. Use technology to view the graph. Were your predictions correct? What occurs on the graph at $x = -2$?

Rational numbers can be written as either proper fractions or improper fractions.

10. Describe the difference between proper fractions and improper fractions and write two examples of each.

A rational expression is similar, except that instead of comparing the numeric value of the numerator and denominator, the comparison is based on the *degree* of each polynomial. Therefore, *a rational expression is proper if the degree of the numerator is less than the degree of the denominator, and improper otherwise.* In other words, improper rational expressions can be written as $\frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomials and the degree of $a(x)$ is greater than or equal to the degree of $b(x)$.

11. Label each rational expression as proper or improper.

$\frac{(x+1)}{(x-2)(x+2)}$ $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$ $\frac{(x+3)(x+2)}{x^4-4}$ $\frac{x+3}{x+5}$ $\frac{x^3-5x+2}{x-10}$

As we may remember, improper fractions can be rewritten in an equivalent form we call a mixed number. If the numerator is greater than the denominator then we divide the numerator by the denominator and write the remainder as a proper fraction. In math terms we would say:

If $a > b$, then the fraction $\frac{a}{b}$ can be rewriten as $\frac{a}{b} = q + \frac{r}{b}$, where q represents the quotient and r represents the remainder.

12. Rewrite each improper fraction as an equivalent mixed number.

a) $\frac{37}{5} =$

b) $\frac{150}{12} =$

Rational expressions work the very same way. If the expression is improper, the numerator can be divided by the denominator and the remainder is written as a fraction. In mathematical terms, we would say:

$\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

Try it yourself! Label each rational expression as proper or improper. If it is improper, then divide the numerator by the denominator and write it in an equivalent form.

13. $\frac{x^2+5x+7}{x+2}$

14. $\frac{-5x+10}{x^3+6x^2+3x-1}$

15. $\frac{x^2+2x+5}{x+3}$

16. $\frac{3x+8}{x-1}$

In *4.3 Rational Thinking,* when we looked at the graphs of rational functions, we did not consider the case when the numerator of the fraction is greater than the denominator. So, let's take a closer look at the rational function from #13.

16. Let $f(x) = \frac{x^2+5x+7}{x+2}$. Where do you expect the vertical asymptote and the intercepts to be?

17. Use technology to graph the function. Relate the graph of the function to the equivalent expression that you wrote. What do you notice?

18. Let's try the same thing with #15. Let $f(x) = \frac{x^2+2x+5}{x+3}$. Find the vertical asymptote, the intercepts, and then relate the graph to the equivalent expression for $f(x)$.

19. Using the two examples above, write a process for predicting the graphs of rational functions when the degree of the numerator is greater than the degree of the denominator.

READY, SET, GO! Name Period Date

READY

Topic: Connecting features of polynomials and rational functions

Find the roots and domain for each function.

1. $f(x) = (x+5)(x-2)(x-7)$

2. $g(x) = x^2 + 7x + 6$

3. $k(x) = \frac{1}{(x+5)(x-2)(x-7)}$

4. $h(x) = \frac{1}{(x^2+7x+6)}$

5. Make a conjecture that compares the domain of a polynomial with the domain of the reciprocal of the polynomial. *(Note that the reciprocal of a polynomial is a rational function.)*

6. Do the roots of the polynomial tell you anything about the graph of the reciprocal of the polynomial? Explain.

7. Find the y-intercept for #1 and #2. What is the y-intercept for #3 and #4?

SET

Topic: Distinguishing between proper and improper rational functions.

Determine if each of the following is a proper or an improper rational function.

8. $f(x) = \frac{x^3 + 3x^2 + 7}{7x^2 - 2x + 1}$

9. $f(x) = x^3 - 5x^2 - 4$

10. $f(x) = \frac{3x^2 - 2x + 7}{x^5 - 5}$

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11. $f(x) = \frac{x^3+4x^2+2x}{10x+7}$

12. $f(x) = \frac{5x^2-4x+4}{7x^5-2x+3}$

13. Which of the above functions have the following end behavior?

$$as\ x \to \infty, f(x) \to 0\ and\ as\ x \to -\infty, f(x) \to 0$$

14. Complete the statement:
ALL *proper* rational functions have end behavior that ____________________

Determine if each rational expression is proper or improper. If improper, use long division to rewrite the rational expressions such that $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

15. $\frac{2x^3-7x^2+6}{x-1}$

16. $\frac{(x+1)}{(x-2)(x+2)}$

17. $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$

18. $\frac{x^3-5x+2}{x-10}$

GO

Topic: Finding the domain of rational functions that can be reduced

State the domain of the following rational functions.

19. $y = \frac{(x-2)}{(x-2)(x+5)}$

20. $y = \frac{(x+6)}{(x-4)(x+6)}$

21. $y = \frac{(x-7)(x+10)}{(x+10)(x-3)(x-7)}$

a) Each of the previous functions has only one vertical asymptote. Write the equation of the vertical asymptote for #19, #20, and #21 below.

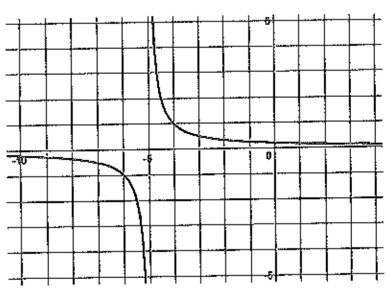
19a) V.A.

20a) V.A.

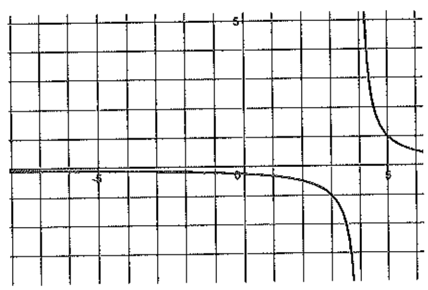
21a) V.A.

b) The graphs of #19, #20, and #21 are below. For each graph, sketch in the vertical asymptote. Put an open circle on the graph anywhere it is undefined.

19b)



20b)



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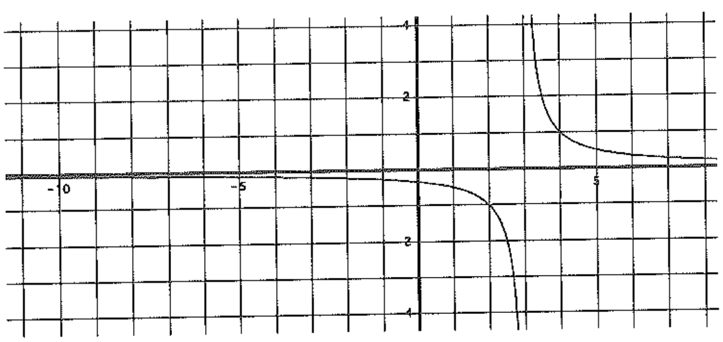

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