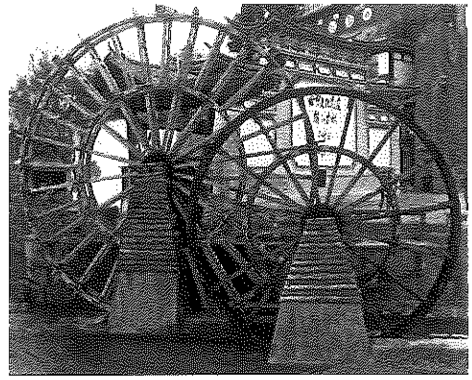
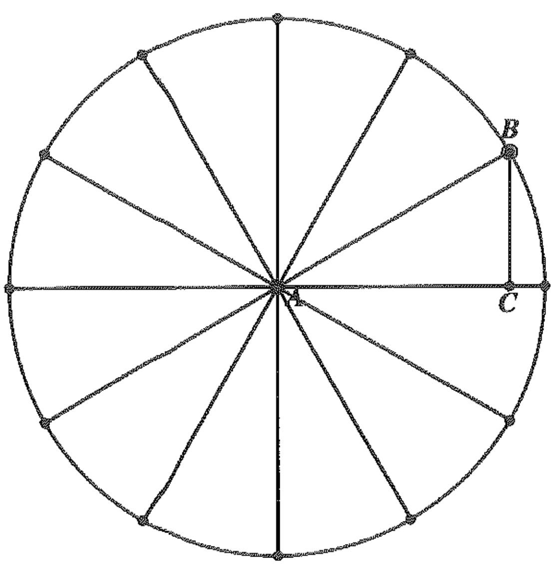
6.9 Water Wheels and the Unit Circle

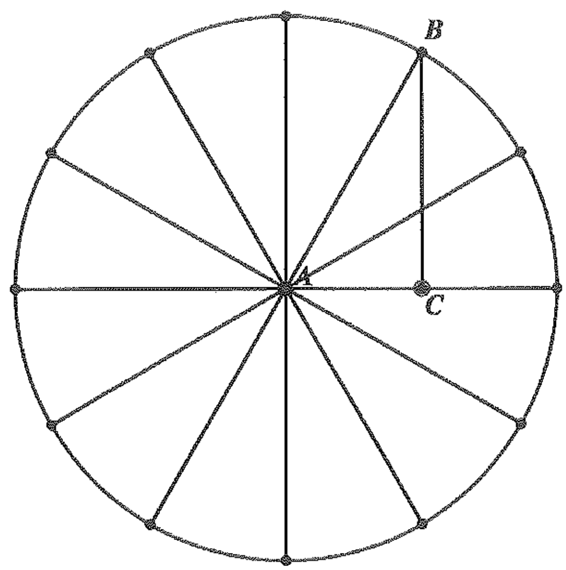


A Practice Understanding Task

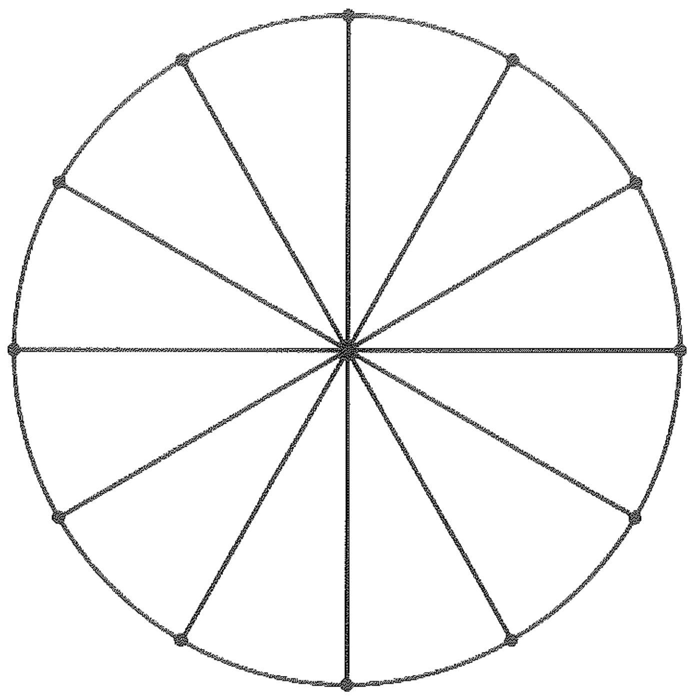
Water wheels were used to power flour mills before electricity was available to run the machinery. The water wheel turned as a stream of water pushed against the paddles of the wheel. Consequently, unlike Ferris wheels that have their centers above the ground, the center of the water wheel might be placed at ground level, so the lower half of the wheel would be immersed in the stream.

1. The following diagrams show potential designs for a water wheel. Each of the 12 spokes of the water wheel will measure 1 meter. In addition to the spokes, the designer wants to add braces to provide additional strength. Two potential placements for the braces are shown in the following diagrams. (The braces and the spoke to which they are attached form a right angle.)
   - Find the measures of $\angle BAC$ and $\angle ABC$ in each diagram.
   - Find the <u>exact lengths</u> of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, not just decimal approximations. Explain how you found these lengths exactly.
   - Label the exact coordinates of point B in each diagram.





2. Based on your work above, label the exact values of the x and y-coordinates for each point on the following schematic drawing of the water wheel. Remember that the center of the wheel is at ground level, so points below the center of the wheel should be labeled with negative values. As in the Ferris wheel models, label points to the left of center with negative coordinates also.



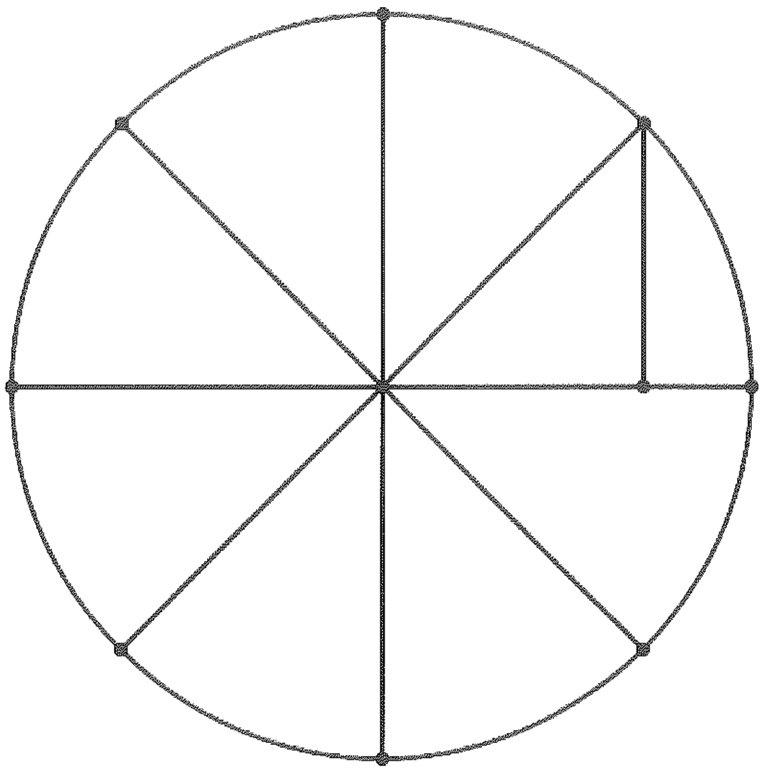
3. Use the diagram above to give exact values for the following trigonometric expressions.

a. $\sin\left(\frac{\pi}{6}\right) =$ b. $\sin\left(\frac{5\pi}{6}\right) =$ c. $\cos\left(\frac{7\pi}{6}\right) =$

d. $\sin\left(\frac{\pi}{3}\right) =$ e. $\cos\left(\frac{\pi}{6}\right) =$ f. $\cos\left(\frac{11\pi}{6}\right) =$

g. $\sin\left(\frac{3\pi}{2}\right) =$ h. $\cos(\pi) =$ i. $\sin\left(\frac{7\pi}{3}\right) =$

4. Here is a plan for an alternative water wheel with only 8 spokes. Label the exact values of the x and y-coordinates for each point on the following schematic drawing of the water wheel. (Hint: You might want to begin this work by finding the length of the "brace" shown in the diagram.)



5. Use the diagram above to give exact values for the following trigonometric expressions.

a. $\sin\left(\frac{\pi}{4}\right) =$ b. $\sin\left(\frac{5\pi}{4}\right) =$ c. $\cos\left(\frac{3\pi}{4}\right) =$

d. $\cos\left(\frac{\pi}{4}\right) =$ e. $\cos\left(-\frac{\pi}{4}\right) =$ f. $\sin\left(\frac{7\pi}{4}\right) =$

g. $\sin\left(\frac{3\pi}{2}\right) =$ h. $\cos\left(\frac{3\pi}{2}\right) =$ i. $\sin\left(\frac{11\pi}{4}\right) =$

During the spring runoff of melting snow the stream of water powering this water wheel causes it to make one complete revolution counterclockwise every 3 seconds.

6. Write an equation to represent the height of a particular paddle of the water wheel above or below the water level at any time t after the paddle emerges from the water.

   - Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

   - Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

During the summer months the stream of water powering this water wheel becomes a "lazy river" causing the wheel to make one complete revolution counterclockwise every 12 seconds.

7. Write an equation to represent the height of a particular paddle of the water wheel above or below the water level at any time t after the paddle emerges from the water.

   - Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

   - Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

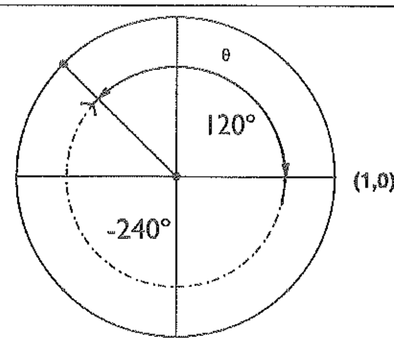
READY, SET, GO! Name Period Date

READY

Topic: Identifying coterminal angles

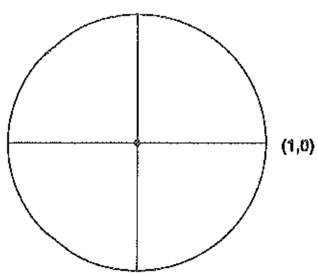
State a negative angle of rotation that is *coterminal* with the given angle of rotation. (*Coterminal* angles share the same terminal side of an angle of rotation.) Sketch and label both angles.

Example: $\theta = 120°$ is the given angle of rotation. The angle of rotation is indicated by the solid arc. The dotted angle of rotation is a coterminal angle with a rotation of $-240°$.



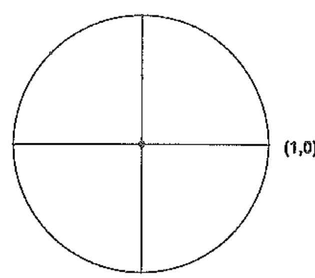
1. Given $\theta = 20°$

Coterminal angle ______________________



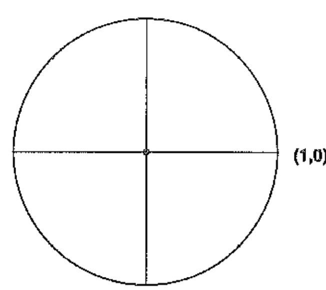
2. Given $\theta = 95°$

Coterminal angle ______________________



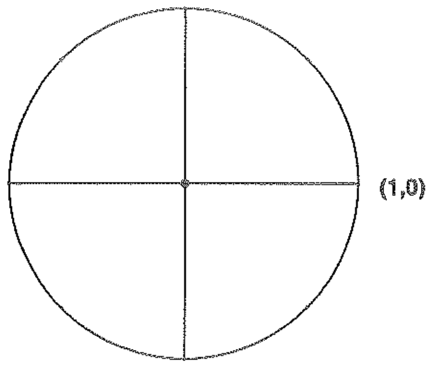
3. Given $\theta = 225°$

Coterminal angle ______________________



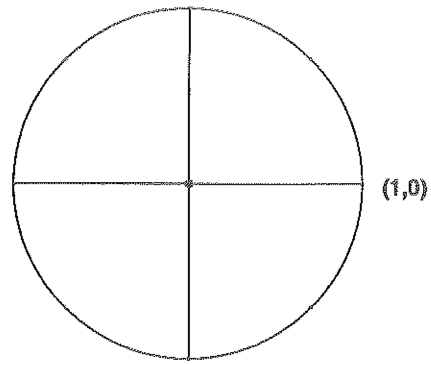
4. Given $\theta = 270°$

Coterminal angle ____________________



5. Given $\theta = 300°$

Coterminal angle ____________________



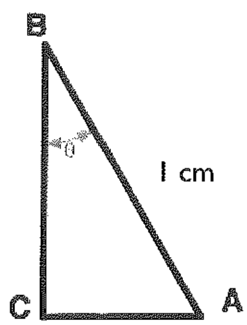
6. What is the sum of a positive angle of rotation and the absolute value of its negative coterminal angle?

7. Every angle has an infinite number of coterminal angles both positive and negative if the definition is extended to angles of rotation greater than 360°. For example: an angle of 45° is coterminal with angles of rotation measuring 405°, 765° *etc.* Given $\theta = 115°$, name 3 **positive** coterminal angles.

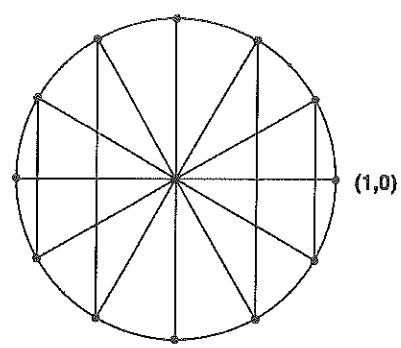
SET

Topic: Calculating sine and cosine of radian measures

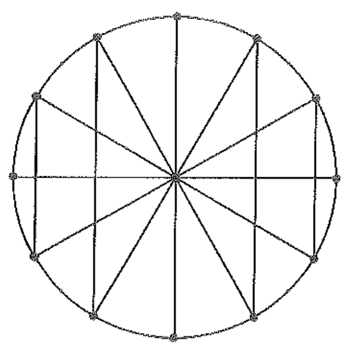
8. Triangle ABC is a 30°, 60°, 90° right triangle. The length of one side is given. Fill in the values for the missing sides. $m\angle B = 30°$.



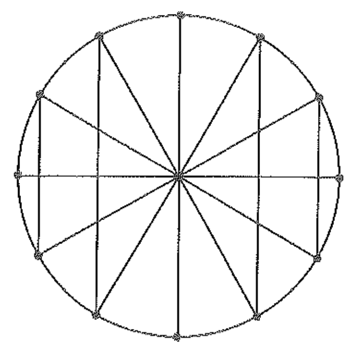
9. Label each point around the circle with the angle of rotation in radians starting from the point (1,0).



10. Use the values in #8 to write the exact coordinates of the points on the circle below. Be mindful of the numbers that are negative.



11. Find the arc length, s, from the point (1,0) to each point around the circle. Record your answers as decimal approximations to the nearest thousandth.



Use your calculator to find the following values.

12. $\sin\frac{5\pi}{6} =$

13. $\sin\frac{\pi}{3} =$

14. Why are both of your answers positive?

15. $\cos\frac{2\pi}{3} =$

16. $\cos\frac{4\pi}{3} =$

17. Why are both of your answers negative?

18. $\sin\frac{\pi}{2} =$

19. $\cos\frac{\pi}{2} =$

20. In which quadrants are sine and cosine both negative?

21. Name an angle of rotation where sine is equal to -1.

22. Name an angle of rotation where cosine is equal to -1.

GO

Topic: Finding the angle when given the trig ratio

Use your calculator to find the value of θ *where* $0 \leq \theta \leq 90°$. Round your answers to the nearest degree.

23. $\sin\theta = 0.82$

24. $\cos\theta = 0.31$

25. $\cos\theta = 0.98$

26. $\sin\theta = 0.39$

27. $\sin\theta = 1$

28. $\cos\theta = 0.71$